Compliments of Harold

ON SENSATIONS FROM PRESSURE AND IMPACT

WITH SPECIAL REFERENCE TO THE INTENSITY AREA AND TIME OF STIMULATION

BY

HAROLD GRIFFING, A.B.

SUBMITTED IN PARTIAL FULFILMENT OF THE REQUIREMENTS
FOR THE DEGREE OF DOCTOR OF PHILOSOPHY
IN THE
UNIVERSITY FACULTY OF PHILOSOPHY
COLUMBIA COLLEGE

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INTRODUCTION.

The extent to which mental phenomena can be measured is not the least important of the many problems before Experimental Psychology. If one mental process is functionally related to another, it is possible for Psychology to become an exact science. If, however, the only measurable attribute of Mind is Time, Psychology can never hope to attain to the exactness of the physical sciences.

The solution of this problem will be found only by experience. The psychologist should not, moreover, be discouraged because Herbart's heroic attempt to apply to Psychology the methods of Mechanics was an ultimate failure, nor yet because Fechner's famous logarithmic law is not now generally accepted. Even if the measurement of mental relations be yet an open question, exact methods may be applied to the investigation of the subjective correlatives of measurable physical phenomena. The most obvious problem of the kind is the relation between the intensity of stimulation and the corresponding sensation. But stimuli may vary in the time and area of application as well as in the intensity. If intensity be a measurable attribute of sensation, and if the time and area of stimulation be also related to the intensity of sensation, the relation of the four quantities may be expressed in the form of an equation:

$$S = f(i, a, t).$$

Only when such an equation is determined will the foundation be laid for the mathematical investigation of mental phenomena. For it is doubtful if exact methods can be applied to the study of mental relations, independent of physical phenomena, until the much simpler problems of Psycho-physics have been solved.

In the following pages we will discuss systematically the relations existing between the intensity, area and time of dermal stimuli, and the resultant sensations and perceptions. We will first, however, treat of dermal sensations with reference to the quality of the stimulus. In this way we shall be in a position to appreciate more fully the significance of the effects of variations of the stimulus in quantity.

CHAPTER I.

Dermal Sensations and the Quality of the Stimulus.

Sec. 1. *Semi-Organic Sensations and their Stimuli.*

Unlike the end-organs of the other senses, that of touch shows traces of the primitive sensibility of the entire periphery. Instead of being specialized in structure and function, the skin has many different and independent functions. Even its sensory functions are quite distinct. Not only are tactile and temperature sensations utterly disparate, but equally distinct are many obscure sensations which, though apparently of dermal origin, seem allied in their vagueness and diffusiveness rather to the group of general or organic sensations. These may be called *semi-organic* sensations, since they represent the transition stage from those sensations which refer to the outer world and those which refer only to the activities of the organism. Nevertheless, we are not justified in considering all dermal sensations as members of the organic group, as has been attempted by some. For temperature and pressure sensations are clearly the data for cognitions of the environment and not of the activities of the organism.

In the case of many of these sensations the stimuli are clearly some peripheral or other physiological processes independent of external agency. Where the sensation appears to be induced by external stimulus, physiologists have endeavored to explain the quality of the sensation by intermediate processes which are considered the true stimuli in such sensations. Among such processes are irradiation, summation, vaso-motor disturbances, and sympathetic nervous action. The resultant *Mitempfindungen* are considered as the subjective effects of heterogeneous sensory excita-

tions.[1] In the case of the tickle sensation, however, which is induced only by external pressure, such pressure must be considered as the stimulus, since it is the physical antecedent of a sensation. The intermediate neural processes may not, moreover, contribute so much to the quality of the sensation as may functional peculiarities of sensory cells. According to Bronson, indeed, the tickle sensation is a relic of a primitive contact sense which existed long before touch proper, and which is, therefore, closely related to the activities of self-preservation and reproduction.[2]

Another state of consciousness, frequently of dermal origin, is pain. If pain be a sensation, it must belong to the organic or semi-organic group; and, in fact, is so classified by Weber, Funke, Wundt and others.[3] As it is not claimed that dermal pain is caused only by secondary nervous excitations, its relation to the stimulus will be discussed in another chapter.

SEC. 2. *Sensations of Touch and Temperature.*

In spite of the universal agreement that the tactile and temperature senses are utterly disparate, it has been claimed on experimental grounds that sensations of touch and temperature are causally related. Weber found that a cold coin was judged heavier than a warm one;[4] and Szabadföldi found, conversely, that a hot wooden cylinder seemed heavier than one of the temperature of the skin.[5] Szabadföldi experimented only on himself; but Weber's experiments were conclusive, and have been corroborated by Dessoir.[6] This writer questions Szabadföldi's results, but we have confirmed them in the following manner:

[1] Quincke, *Zeit. für Klin. Med.*, Bd. xvii. 1890, 429; Goldscheider, *Berlin. Physiol. Gesell.*, 1890–91, no. 1, 5; Külpe, *Grundriss der Psy.*, 92; Wundt, *Grundzüge der Phys. Psy.*, iv.te Auf., 1, 408; Dessoir, *Archiv. für Anat. und Physiol.*, 1892, 324.

[2] Bronson, *The Medical Record*, xxviii. 425.

[3] Weber, *Wagner's Handbuch der Physiol.*, iii. 2te Abth.; Funke, *Hermann's Physiologie*, iii. 292; Wundt, *op. cit.*, i. 544.

[4] Weber, *op. cit.*, 512.

[5] Szabadföldi, *Moleschott's Untersuchungen*, IX, 624.

[6] Dessoir, *op. cit.*, 305.

A 25-cent silver coin was heated in water to a temperature of from 50° to 55° C., and then placed carefully upon the palm of the observer's hand, the eyes being closed. It was then removed, and a similar coin heated to about the temperature of the skin was placed upon the hand. This was repeated a number of times, though occasionally the hot stimulus was the second to be applied. Four observers judged the hot coin heavier, and one showed no marked constant tendency. With one observer the writer applied two coins simultaneously, one over the other, the pressure of the two being compared with that of the hot coin. The one hot coin was judged heavier five times in ten trials, some of the observer's answers being guesses. From these experiments we conclude that pressure stimuli of low intensity and high temperature are judged heavier than those having the temperature of the skin.

It does not follow, however, that all stimuli thus differing in temperature will give rise to such illusions. In order to ascertain whether hot or cold weights of high intensity are judged heavier, the writer applied to the palm of the hand a brass kilogram weight heated to about 50° C. This was removed and placed again upon the hand, but not in contact, a circular card-board of the area of the base lying between the weight and the skin. The hand of the observer was comfortably supported. Different persons served as subjects, and all were ignorant of the purpose of the experiment. As in the preceding experiment, a number of trials were made for each observer. Similar experiments were made with a cold weight and one which had no appreciable temperature effect on the skin. The cold weight generally had a temperature equal to that of the room, about 20° C., and at times much below this, so that from the area of stimulation, 16 sq. cm., and the conductivity of the metal, a marked sensation of cold was produced. It was found, as shown in the table of results given below, that stimuli of very high or low temperature are not judged heavier at 1000 g. In fact, the hot weight is rather judged lighter. In the table here given the figures denote the number of times the cold and

hot weights were judged heavier or lighter than those of moderate temperature.

	Cold weight, 1 kg		Hot weight, 1 kg	
Observer.	Heavier.	Lighter.	Heavier.	Lighter.
L. F.	2	6	2	8
S. F.	–	–	0	10
K.	2	3	2	5
M. G.	7	3	4	6
J. G.	5	5	4	6
Total,	16	17	12	35

The results above given go to show that tactile and temperature sensations are not related, as Weber[1] and Szabadföldi[2] inferred. Dessoir's explanation is that the illusion is due to the contraction of the skin from the lower temperature, and consequent increase in the number of sensory nerves that are affected.[3] But heated coins are overestimated, and according to this hypothesis they should be underestimated. A more satisfactory explanation is that an illusion of judgment is involved.[4] This is rendered plausible by the fact that stimuli of high intensities are not overestimated. We may suppose that the mind tends to infer from the intensity of the temperature sensation that the corresponding stimulus is of greater magnitude, and therefore heavier than the stimulus causing a purely haptic[5] sensation of but little intensity. For heavy weights we should on this hypothesis expect underestimation rather than overestimation, of hot or cold stimuli, and that there is some such tendency, at least for hot weights, the experiments seem to show. The objection of Dessoir against such an explanation is, we think, inconclusive. A difference in dis-

[1] Weber, *op. cit.*, 551.

[2] Szabadföldi, *op. cit.*

[3] Dessoir, *op. cit.*, 306.

[4] *Cf.* Funke, *op. cit.*, 321.

[5] We use the term *haptic* (Greek ἁπτάομαι) of all sensations of contact, touch, pressure or impact. For this term we are indebted to Dessoir.

crimination-time for weight and temperature when only the quantity judged is variable, does not preclude such an illusion when the conditions are different.

The other experimental evidence in favor of any fundamental relation between haptic and temperature sensations is equally inconclusive. The fact that heavy weights seem hotter or colder than lighter weights, as stated by Nothnagel,[1] may be due to differences in conduction arising from differences in contact. Wunderli found that observers had difficulty in distinguishing tactile from temperature stimuli of low intensity.[2] But the errors occurred only when the back was the surface stimulated, and though temperature stimuli were confused with tactile stimuli, the reverse error did not occur. As we are not accustomed to temperature sensations in the back, such a confusion is but natural, especially when the stimuli are of such low intensity that the process of perception is obscured.

SEC. 3. *Active Touch.*

The great majority of so-called tactile sensations are in reality results of complex kinaesthetic and haptic sensory elements. In fact, many have distinguished between active and passive touch. Dessoir opposes contact sensations to those of pselaphesia,[3] and Bronson goes so far as to consider contact sensations and those of active touch not only as quite distinct but as having different end organs.[4] It is clear, however, that active touch may involve movement with or without muscular effort, or, conversely, muscular effort with or without movement. We have, therefore, a triple set of sensory impulses to consider as the physiological antecedents of the sensation of active touch.

Many psychologists have explained the sensation of movement by alterations in the tension of the skin and by atmospheric pressure.[5] This view is apparently corroborated

[1] Nothnagel, *Deutsches Archiv. für Klin. Med.*, II, 298.

[2] Wunderli, *Moleschott's Untersuchungen*, VII, 393.

[3] Dessoir, *op. cit.*, 242.

[4] Bronson, *op. cit.*

[5] For references see the works of Wundt and James, and Delabarre, *Ueber Bewegungsempfindungen*, Freiburg, 1891.

by the influence of dermal anaesthesia or hyperaesthesia on the perception of movement. The pathological evidence proves that dermal sensations enter into those of movement, but that is all. Other well authenticated observations show that anaesthesia does not necessarily affect the perception of movement. In complexes of tactile and kinaesthetic sensations we must, therefore, assume different sensory processes.

But active touch is possible without movement either of the stimulus or the sense organ. If through an act of volition we exert pressure upon an external object, we have in addition to the sensation of dermal pressure that of effort. In fact, all the feelings of strain and tension are felt which enter into the muscular consciousness. As pathological observations and experiments on lifted weights prove the muscular sense to be independent of touch,[1] it is evident that where pressure is exerted voluntarily the resultant sensation is complex, and not a haptic sensation proper. We have, therefore, to distinguish between what we might call subjective pressure, or pressure with effort, and objective pressure, or pressure without effort.

SEC. 4. *Passive Touch.*

Having analysed the various elements entering into tactile complexes, we turn to those sensations in which the subject is passive and the stimulus acts only upon a definite area. The stimulus may then be pressure exerted upon the skin, the energy of a body striking the skin, or traction tending to separate the dermal end organ from the organism to which it belongs. These stimuli are qualitatively different, as are the corresponding sensations, though for traction these are less distinct than would be supposed.[2] The blow of a moving object upon the periphery gives rise to a sensation distinct from that of a motionless weight. This difference increases with the velocity of the moving mass. The stimulus in such sensations, therefore, is to be considered the product of the mass and its velocity, or some function of its velocity. The resulting sensation may be called a sensation of impact, as distinguished from one of pressure.

[1] See Wundt, *op. cit.*, 427; Delabarre, *op. cit.*, 37, 38.

[2] See Hall and Motora, *Am. Journal of Psy.*, I, 72.

But is not this difference between pressure and impact only a difference in degree? When a weight is applied to the hand there must be some impact, whatever be the velocity at which the weight be applied. If a weight of low intensity, as 100g or less, be applied, and the area of stimulation be not too small, the sensation is one of impact; but if a stimulus of moderate intensity be used, a distinct pressure sensation will be observed in addition to that of impact. This is due to the effect of the weight in overcoming the elasticity of the skin and depressing the dermal tissues. The stimulus in pressure sensations may, therefore, be considered not momentum or kinetic energy, but rather as mechanical force exerted through the object in contact with the skin, or more accurately, the work done by this force in displacing the dermal tissues. In reality, however, the process of stimulation is often more complex. When the pressure is sufficiently great to produce motion, kinaesthetic elements will affect the sensory result. When movement is prevented by opposing forces, a double process of dermal stimulation will result, since action and reaction are equal and opposite.

Sec. 5. *The Classification of Dermal Sensations.*

The general results of the analysis above given may be summarized in a classification of dermal sensations with special reference to the quality of the stimulus:

- Dermal Sensations
  - Simple, or Purely Dermal
    - Haptic
      - Traction.
      - Objective pressure.
      - Impact.
    - Temperature
      - Heat.
      - Cold.
    - Semi-organic
      - Tickle.
      - Itching.
      - Creeping.
      - Pain.
  - Compound, or Semi-Dermal
    - Subjective Pressure
      - With movement.
      - Without movement.
    - Kinaesthetic
      - With effort.
      - Without effort.

CHAPTER II.

The Intensity of Stimulation.

Sec. 1. *The Concept Intensity.*

The term intensity, as applied to neural stimuli, has long been in universal use among psychologists, but frequently in a manner that is far from exact. In physical science the term is used as the quantitative predicate of force. But many stimuli, as those of smell and taste, cannot be measured in terms of force. To avoid ambiguity in the use of this term, we would suggest as a working definition of intensity, as used in Psychology, the following, which is based upon obvious psychological grounds: that quantitative property of neural stimuli, the magnitude of which determines whether or not they give rise to a sensation; and, if so, whether that sensation be painful, or have only the particular quality due to the quality of the stimulus.

From this point of view the intensity of visual and auditory stimuli may be measured by the energy of motion transmitted to the end organ in a given time. The intensity of gustatory and olfactory stimuli may be measured for a given substance by the quantity which is applied to the end organ. But with temperature stimuli the measure of intensity is more complex. Then, too, as heat and cold are physically the same, the absolute measure of heat is not the measure of the intensity of heat, as regards its physiological and psychological effects. Passing from the temperature sense to that of effort, the work done by muscular contraction in a given time is clearly the measure of intensity; but when no motion takes place the criterion is different, as such units cannot be used. In this case the measure of intensity is clearly the force which is exerted.

The measurement of the intensity of haptic stimuli is, fortunately for our purpose, comparatively simple. When

impact may be neglected, the intensity of the stimulus is measured by the weight that is applied; for the work done in depressing and displacing the dermal tissues will be proportionate to the impressed force. When, however, appreciable movement occurs before the full pressure is exerted, the matter is more complex, since the subjective effect is dependent not only on the mass but also on its velocity. We might suppose that the measure of the intensity of an impact stimulus would be the product of the mass and the square of the velocity, since this quantity represents the energy of the blow. But, as we shall find in the chapter on Sensations of Impact, the square of the velocity does not appear to have as intensive an effect as does the mass.

SEC. 2. *Touch and Pressure.*

It was shown by Aubert and Kammler that pressure and impact stimuli, below a certain intensity, are not perceived.[1] The sensations from stimuli of low intensity are sensations of passive touch, the element of pressure being apparently absent. From such data Meissner inferred that pressure sensations are absolutely distinct from those of touch proper, *einfache Tastempfindungen*, and that these have special end organs, the tactile corpuscles.[2] Meissner's distinction between touch and pressure is accepted by Aubert and Kammler, Bronson and Dessoir, but is rejected by Funke, Wundt and Külpe. We shall now consider in detail the evidence that has been brought forward to support this view.

According to Meissner, touch furnishes the data for the concept of externality, and accompanies all pressure sensations, though not necessarily accompanied by them.[3] Clearly, however, this is but an hypothesis to account for what is assumed, that is, the difference between touch and pressure. Aubert and Kammler reject Meissner's hypothesis, basing their distinction upon their alleged observation that contact sensations are subjective modalities. This does not accord

[1] Aubert and Kammler, *Moleschott's Untersuchungen*, v. 145.

[2] Meissner, *Zeitschrift für Rat. Med.*, 2te R., iv.; also, *Beiträge zur Anatomie und Physiol. der Haut*, Leipzig, 1853.

[3] Meissner, *op. cit.*, 272.

with the introspection of the writer. The apparent difference observed may be due to the fact that stimuli of low intensity do not give rise to sensations of sufficient clearness for the mind to perceive the quality of the stimulus. We certainly do refer a tactile sensation to an external *something*, though what that may be we may not know.

Dessoir gives as the characteristic of pressure sensations the feeling of effort which is involved.[1] But Dessoir undoubtedly refers to subjective pressure, or pressure with effort; and as sensations of pressure are possible without effort, the criterion is not applicable.

Bronson bases his separation from pure contact of *pselaphesia*, or perceptive touch, partly upon the above facts of introspection and partly upon the apparent relationship of sensations of contact to semi-organic sensations.[2] According to Bronson, sensations of contact require as their peripheral antecedents only the stimulation of the epidermal fibrillæ, and are, therefore, to be considered distinct and primitive sensations.

However conclusive be Bronson's arguments as to the biological theory of dermal sensations, they do not prove touch to be distinct from pressure, because the tickle sensation does not necessarily accompany that of contact. It is a distinct state of consciousness independent of the tactile sensation, and the same may be said of the aphrodisiac sense. We conclude, therefore, that there is no psychological basis for the distinction, unless there be other evidence than that which we have discussed. If touch and pressure were distinct, we should look for such evidence in pathology; but the writer knows of none. Bronson states that hyperaesthesia and apselaphesia may coexist. But it is probable that he really refers to hyperalgesia, which is quite irrelevant. According to Richet, tactile hyperaesthesia is unknown.[3] There have been instances of anæsthesia for pressure stimuli of low intensity without anæsthesia for those of high inten-

[1] Dessoir, *op. cit.*, 242.

[2] Bronson, *op. cit.* Bronson does not state these arguments categorically, but the above appears to be his position.

[3] Richet, *Récherches sur la Sensibilité*, 219.

sity.[1] But, as Richet observes, this may be explained by the fact that the nerves die first at their extremities.

Apart, however, from these negative considerations, it must be admitted that the classification of one group of sensations, as distinct from another group, logically implies our inability in introspection to pass gradually from one to the other. By this criterion the sense of touch and that of pressure must be identical. It is impossible to tell where one begins and the other ends. Stimuli that are barely perceptible may be judged with reference to their weight.[2] On the other hand, individuals differ as to what they call pressure. In the course of experiments on the threshold of pain, to be described in the next section, one observer said he began to feel pressure at 3.5k., pain appearing at 8.5k. The writer would call that sensation one of pressure when the instrument used registered only 1.0k.

Even if touch and pressure be indistinguishable, the apparent change of quality requires an explanation. That generally given is that different physiological processes are induced by intense stimuli. Aubert and Kammler explain the distinction by the displacement of the skin. But this displacement varies with the intensity of the stimulus.[3] Külpe mentions the effect of intense pressure upon the muscular tissues,[4] but we have pressure sensations where there are no muscles. Meissner's hypothesis, to which that of Bronson is similar, that the sensory cells in the dermis are the anatomical basis of pressure sensations, is inadequate, since these cells appear to be absent on parts that are sensitive to pressure.[5] Goldscheider found special pressure spots,[6] but his results, both histological and psychological, are disputed.[7] The writer's own observation does not enable him to detect the existence of points that give pressure or contact sensations only. Certain spots may be more sensitive

[1] Richet, *op. cit.*, 227.

[2] See Chapter III., Section 3.

[3] For measurements of this, see Hall and Motora, *op. cit.*

[4] Külpe, *op. cit.*, 91.

[5] Cf. Wundt, *op. cit.*, i. 302; Dessoir, *op. cit.*, 275.

[6] Goldscheider, *Archiv. fur Anat. und Physiol.*, 1885 Supp. Bd., 76.

[7] Cf. Dessoir, *op. cit.*, 251.

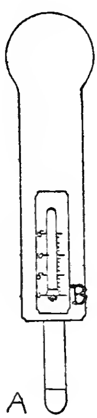
than others, but this would throw no light on the question. Besides, it is difficult to obtain a distinct sensation of pressure when so small an area is stimulated as is necessary in such experiments, since the sensation of pressure passes so quickly into that of pain or other semi-organic sensations.

But if there is no additional process of sensory excitation in pressure sensations, in what way may the apparent difference be explained? Our answer is that there is no difference in sensation, but only in perception. What we mean by a sensation of pressure is one of such a quality that we can ascribe the subjective effect to some definite objective cause and one exerting such pressure that its removal would involve appreciable muscular work. The apparent difference may, we think, be thus explained, for it is impossible to analyse in consciousness the mental reaction in perception out of the total sensational and perceptive complex.

3. *The Threshold of Pain.*

For the purpose of measuring the intensity of pressure causing pain a spring dynamometer was used by which a given pressure could be exerted upon any surface.[1]

FIG. 1.



Attached to the lower end of the spring was a sliding cylindrical piece of brass. This was capped with hard rubber (A), which was applied to the surface to be stimulated. The cap which came in contact with the skin was hemispherical, and about 8mm. in diameter. The pressure was exerted by the hand of the experimenter, and the amount of pressure was registered in kilograms by the movable piece (B) attached to the spring. The scale was tested by an accurate balance adapted to heavy weights, and was found to be free from appreciable error. The stimulus was applied by the writer to the volar surface of the left hand of the subject over the fifth meta-

[1] This instrument was devised by Prof. J. McK. Cattell. He has suggested the term *algometer* by which to designate it, and this expression will be used hereafter.

carpal. The pressure was increased as nearly as possible at the same rate for different observers, about 1.4k. per sec. If we take .3 sec.[1] as the double reaction-time, we have to subtract $1.4 \times .3 = .4$k. from the reading of the instrument. The observers were asked to speak when the instrument began to hurt at all or be uncomfortable; for it was found that individuals differed as to what they called 'pain.' The subjects tested were students in Columbia and Barnard Colleges and in private schools.[2] Below we give the average in kilograms as well as the maxima and minima corrected for the constant error above referred to. The approximate ages are also given.

Observers.	50 Boys.	40 College Students. (Men.)	38 Law Students.	58 Women.	40 College Students. (Women.)
Ages,	12 to 15	16 to 21	19 to 25	16 to 20	17 to 22
Av.,	4.8	5.1	7.8	3.6	3.6
Max.,	8.4	13.6	15+	7.6	8.6
Min.,	2.1	1.9	3.9	1.8	1.7

From the above results it appears that although individuals differ greatly in sensitiveness to pain, on the whole women and boys are more sensitive than men. The variations in those of the same age and sex are not due to chance, since any one person when tested gives fairly constant results. Nor are they due to individual differences in perception and judgment, though doubtless these affect the results to some extent; for it is very easy to tell when the pressure begins to be uncomfortable, and the 'imagination' does not seem to be a disturbing factor. Indeed, the pain seems often to come with greater suddenness. These variations are rather to be ascribed to constitutional nervous differences, and in part, perhaps, to differences in the thickness of the skin.

[1] This was verified by chronoscopic measurements.

[2] The writer takes pleasure in acknowledging his indebtedness to Registrar Mrs. N. F. Liggett and Principals Miss Brown and Mr. Cutler for furnishing him the opportunity of making the tests on young women and boys.

SEC 4. *The Range of Pressure Sensations.*

If the *minimum tangible*, or tactile threshold (T), were measurable, as is the threshold of pain (P), and if the sensation of pressure ceased as soon as that of pain appeared, we could determine the range of haptic sensations (R) by the formula:

$$R = \frac{P}{T}.^{1}$$

Since the haptic sensation does not cease when pain begins, but rather decreases gradually as the pain increases, the so-called range cannot be measured. We may, however, use the term to indicate the extent of haptic sensations up to the pain threshold. But are we justified in assuming the pain threshold to be a quantity? According to the algedonic[2] tone theory we are not so justified. And, even assuming that a stimulus becomes painful at a certain point, the one sensation is at first so obscured by the other that it is not immediately appreciable. Nevertheless, the appearance of pain is generally so sudden when the stimulus is increasing in intensity, that we treat the threshold of pain as approximately a quantity.

Assuming then that the range, and therefore the thresholds of touch and pain, can be measured, it is evident that in determining them the conditions of stimulation should be constant. Not only the time and space conditions, but also the mode of applying the stimulus, must be constant.

In the measurements of the tactile threshold made by Aubert and Kammler the element of impact was involved, and their results could not be compared with our own measurements of the pain threshold, since in these impact was excluded. Bloch employed the same method, that of pure pressure,[3] but his experiments were made on himself, and we judge them, therefore, inconclusive. We found that results are obtained under such circumstances quite different from those obtained when the stimulus is applied by another

[1] *Cf.* Wundt, *op. cit.*, I, 335.

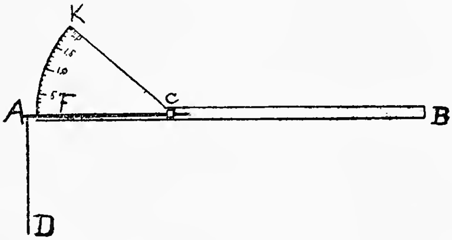
[2] We have borrowed this translation of *Gefühlston* from Marshall, *Pain, Pleasure and Æsthetics*.

[3] Bloch, *Archives de Physiologie*, 1891, 322.

person and the observer is ignorant of the time of application. Bloch gives .0005g to .0015g as the smallest appreciable pressure. Aubert and Kammler found for an area of 9mm, .005g as the *minimum tangibile.* The results above given are much more discordant than they might at first seem, since the greater value for the threshold is obtained for the smaller area,[1] and since impact is clearly involved rather than pressure.[2]

In order to make further experiments on the smallest perceptible haptic stimuli, the writer constructed an instrument similar to that used by Bloch.

FIG. 2.



To a wooden handle (B) was attached by wax a horizontal bristle (AC), taken from an ordinary broom. At the end (A) was fastened by wax a vertical piece (AD) of the same material, the point of which was applied to the part stimulated. The pressure was exerted by the hand of the experimenter. The degree of pressure was shown by the elevation of the bristle, which was read off on a scale (FK). The readings on this scale were in grams, the elevations corresponding to different pressures having been found by a balance. The pressure was applied upon a circular card board about .9cm in area. This card was so light, .05g, that its weight could be neglected after the moment of application, as it rested on the skin during the experiments. The observer's eyes were closed, and he did not know when the stimulus was applied. The rate of application of the pressure was kept as constant as possible. It was as rapid as was consistent with taking the readings, about .3g per sec.

[1] See chapter V, sec. 2.

[2] See chapter VI, sec. 2.

We subtract, therefore $.3 \times .3 = .1$g from the reading obtained. Ten experiments were made on S. F., and also on G., the writer, for the smallest perceptible pressure, T. The same number were made for the threshold of pain, P, by means of the dynamometer already described. The area of stimulation for the pain measurements was also .9cm. The results are given in grams. The values of the Range, R, are found by dividing the average values of P by the average values of T.

	T.			P.			$R = \frac{P}{T}$
	Av.	Max.	Min.	Av.	Max.	Min.	
F.	1.9	2.7	1.	3230	4300	2700	1700
G.	2.6	2.5	.4	4400	5700	3800	1697

According to these results the haptic range is about 1700. The great variation in the values obtained for the threshold renders these figures necessarily very inexact. The values of the threshold here given are very much greater than those obtained by previous investigations. The elimination of the element of impact[1] and of the knowledge of the observer would tend to give far greater values than those obtained by Bloch and by Aubert and Kammler. Then, too, the area and time of stimulation are factors not to be neglected; but these differences are not such as to affect the results appreciably.[2]

It is generally assumed that the threshold is a definite quantity.[3] In the case of sensations of pain, the results obtained for any individual are sufficiently constant to justify this assumption as a working hypothesis. The results given above for the tactile threshold are, however, so variable that we are led to doubt the validity of such an assumption. In fact, the very conception of a threshold involves a logical contradiction. If by this we mean a quantity that we can always perceive under moderately constant conditions of

[1] See Chapter V, Sec. 1.
[2] See Chapter VI, Sec. 2; Chapter VII, Sec. 1.
[3] Wundt, *op. cit.*, I, 334; Külpe, *op. cit.*, 51; Ladd, *Elements of Phys. Psy.*, 363.

attention, we shall have to assume a quantity much larger than what we often perceive. In the course of experiments on the perception of differences in weights, the application of a stimulus of 5g was unobserved several times, and that, too, by an excellent subject, who was expecting the stimulus at the time of application.[1] Even a stimulus of 100g has been unobserved by good observers in experiments by the method of right and wrong cases. We must conclude, then, that stimuli of a given intensity will be observed a certain proportion of times and no more, if a sufficient number of experiments be made. We may also infer that stimuli far below the so-called threshold will be observed, some times, at least, in an infinite number of trials. What, then, shall we call the threshold? It is not the quantity that is always observed, for this would involve a contradiction. It is not that which is observed a certain percentage of trials, for this could not be called the least perceptible intensity. We can only say that the probability that a given stimulus will be perceived by the observer is functionally related to the intensity of the stimulus. In fact, the so-called threshold is no more a definite quantity than the so-called least noticeable difference, which we think leads, when discussed from the standpoint of probabilities, to a similar *reductio ad absurdum.*[2] Indeed, the processes involved are much the same. Not the least important of the factors entering into the measurement of one as well as the other, is the confidence of the observer, which varies from extreme doubt to absolute certainty.[3] The wrong cases, or mistakes due to errors of observation, which occur when different stimuli are compared, have their counterpart in tactile hallucinations, a number of which occurred in the course of our experiments on the threshold.[4]

[1] See Chapter III, Secs. 2 and 3.

[2] Fullerton and Cattell, *On the Perception of Small Differences*, 10; Pierce and Jastrow, *National Academy of Sciences*, 1884, III, 75.

[3] See Chapter III, Sec. 5.

[4] *Cf.* Krohn, *Journal of Mental and Nervous Diseases*, March, 1893, 14.

SEC. 5. *The Intensity of Sensation and the Intensity of the Stimulus.*

This relation has generally been investigated by deductions from the relation of the least noticeable differences to the absolute intensity of the stimulus. But, as is shown by the application of the method of right and wrong cases, there is no such quantity, and therefore the deductions based upon it are invalid. Of the other psycho-physical methods, two have been applied by Merkel to the investigation of haptic sensations.[1] By the method of double stimuli it was found that the ratio of the normal to the estimated double stimulus was approximately 1 : 2 for from 100g to 2000g. By the method of mean gradation Merkel found that the values of the estimated arithmetic mean of two stimuli was but slightly less than the true arithmetic mean. Merkel's experiments were, however, made only on himself, and the muscular sense was not excluded, so that his results are not conclusive.

In the hope of throwing some light on the much discussed psycho-physical problem in pressure sensations, experiments were made by a method different from those generally used, the observer being required to judge of two stimuli how much greater one was than the other. The method of experiment in detail was as follows. A wax mould having been constructed to fit the left hand, the hand was placed in this, the palm being upward under the pan of a balance. The pressure was given by weights placed upon the pan of the balance. The pressure was transmitted to the hand by means of a piece of wood glued to the pan. A circular cap of cardboard attached to the end of the stick, and about 4 mm in diameter, came in contact with the skin. The observer having closed his eyes, and the cardboard cap being barely in contact with the skin, a weight was carefully placed in the pan, and after about two seconds was removed and replaced by a weight very much heavier, the observer being asked to judge the ratio of the weights. But few experiments were made at one sitting, so that memory

[1] Merkel, *Philosophische Studien*, v. 253.

could not affect the results. For purposes of convenience the lowest stimulus was applied first, the next higher following; but the reverse order was at times adopted without perceptible difference. The observers were, of course, ignorant of the objective relations of the weights as well as of the purpose of the experiment. They were all students of Psychology. In the table appended are given the results. The first horizontal column denotes the stimuli in grams. The numbers in the vertical columns, under those denoting the stimuli, indicate the average judgments as to how many times the given stimulus was greater than the stimulus preceding. Thus S. F. judged 50 g., 3.1 times as heavy as 10 g, and 250 g., 4.2 times as heavy as 50 g. The figures preceded by the sign $\pm$ denote the probable error of the given average.[1]

But few experiments were made on each observer, for not only were the mean variations small, but the individual differences were very great.

Observer.	No. expts.	2 g.	10 g.	50 g.	250 g.	1250 g.	1800 g.
S. F.	10×4	—	—	3.1±.01	4.2±.03	7.1±.04	3.8±.04
L. F.	6×4	—	—	2.0±.00	2.7±.00	4.9±.01	3.9[2]±.01
P.	5×5	—	2.2±.00	2.5±.01	3.0±.00	5.6±.03	3.4±.01
K.	5×5	—	1.9±.00	1.9±.00	2.1±.00	3.4±.00	1.7±.00
Av.	—	—	2.0	2.4	3.0	5.2	3.0[3]

In order to represent the relation between the stimulus and the estimate of the stimulus, let us take the number 2 as representing the estimated weight at 2 g. Multiplying this by the estimated values of 10 g. in terms of 2 g., the number obtained will represent the increase of the estimate of the stimulus as the stimulus increases from 2 to 10. In like manner, by taking this result and multiplying it by the esti-

[1] This is such an error (or deviation from the average) that half of the errors would be smaller and half would be larger. It is here obtained by the briefer formula,

$$P = \frac{.845 \ \Sigma v}{n \sqrt{n-1}}$$

See Merriman, Airy and other writers on the theory of probabilities and the method of least squares.

[2] This number refers to 2500 g., not to 1800 g., as do the others in this column.

[3] This average is based upon 3 values, 3.8, 3.4 and 1.7. See note 1.

mated values of 50 g. in terms of 10 g., we obtain the increase of the estimate as the stimulus increases from 10 g. to 50 g. In thé case of S. F. and L. F., as no measurements were made of the estimate of 10 g. in terms of 2 g., we take as the unit of estimated weight at 10 g. 4.4, which is the value obtained for P., with whose results those of S. F. and L. F. fairly agree.[1] In this way the relative increase of the estimate of weights is obtained. The increase of the stimulus is shown by the intensities used, these being, with the exception of the highest, in geometrical progression. Below are given the calculated values of the estimates of the stimuli:

Observer.	2 g.	10 g.	50 g.	250 g.	1250 g.	1800 g.
S. F.	—	4.4[2]	13.	57.	404.	1535.
L. F.	—	4.4[3]	9.	23.	112.	433.[4]
P.	2	4.4	11.	33.	185.	629.
K.	2	3.8	7.2	15.	51.	86.
Av. estimate of stimulus,	2	4.2	10.	32.	188.	750.[4]

The relations here expressed are graphically represented in the accompanying curves. The ordinates express the estimates of the intensity of the stimulus, and the abscissae the true intensities in grams.

In interpreting these results we may assume that the intensity of sensation increases in proportion to the estimated increase of the stimulus. Such an assumption would be illegitimate, if the stimuli were such that the observers could judge them by some means other than their effect on sensation. But where the muscular sense is excluded, as in these experiments, association cannot very well influence the

[1] 2 g. was found to be so often inappreciable by S. F. that the determination based upon it was difficult. The experiments on L. F. were made before it was decided what weights had best be used.

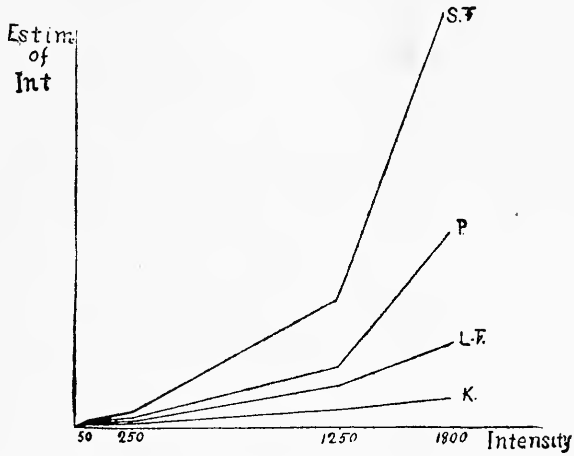
[2] These are taken as units, as explained above.

[3] This number refers to 2500 g. instead of 1800 g.

[4] Based upon three values. See note 3.

results; for the concept of weight is based upon sensations of effort. Assuming, then, that a relation is obtained between the intensity of the stimulus and that of the sensation,

FIG. 3.



it is evident that for moderate intensities the sensation increases much more slowly than in direct proportion to the stimulus. As the stimulus approaches the pain threshold, the sensation appears to increase at a much greater rate than before. The individual variations are so great as to render impossible an analytical expression of the relation. Nevertheless, the shapes of the different curves are similar. It is clear, moreover, that a logarithmic relation as demanded by Fechner's law does not hold, even within narrow limits, for any one of the observers. If such were the case, the estimates of the stimulus would increase arithmetically, since the stimulus increases geometrically.

It is, however, possible that this relatively rapid increase for high intensities is due to processes of perception and judgment, and not to real differences in the rate of increase of the sensation. As stimuli approach the pain threshold, the consciousness of impending pain may cause us to overestimate the magnitude of the stimulus. This might happen in either of two ways. In the first place, since the sensation of pain and that of pressure are heterogeneous, we might suppose that the mind would unconsciously assume great

objective differences in quantity as causally related to subjective differences in quality. Another possible explanation is that the sensation of pain, tending to occupy the field of consciousness to the exclusion of other presentations, is to be considered as essentially a sensation of great intensity; from which it follows that stimuli causing pain, or approaching the pain threshold, are estimated as relatively of greater intensity than those into the perception of which the element of pain does not enter.

SEC. 6. *Haptic Sensations and Dermal Pain.*

We have already found, in Chapter I, that the peculiar quality of the tickle sensation is not logically ascribable to the quality of the stimulus. We may state, therefore, that sensations differing in quality may be caused by stimuli differing in quantity. It is not, however, near the lower limit of haptic stimulation that this qualitative transition is most marked. If pain be considered a sensation, two disparate sensations are induced by high as well as low intensities of dermal stimuli. If, however, pain be considered but an intensive form of an element existing in all sensational states of consciousness, such a generalization is impossible.

According to the commonly accepted view, the algedonic[1] tone of a sensation is negative, that is, unpleasant, for very low intensities, but upon increase in the stimulus becomes positive. As the stimulus is further increased, a maximum of the positive values is reached, after which the algedonic tone rapidly decreases. This doctrine has, undoubtedly, many theoretic advantages. But it does not seem to accord with the observed phenomena of dermal sensation. In the experiments on pain already described, the appearance of pain was generally quite sudden. If the pain consciousness were merely an intensive form of what accompanies all dermal stimulation, we should not expect such sudden transitions. Then, too, in the writer's experience, at least, there is no pleasurable element whatsoever in a haptic sensation of moderate intensity. It may be said that we prefer certain

[1] Wundt, *op. cit.* I, 558; Külpe, *Grundriss der Psychologie*, 256. See also the writings of Ward, Sully, and Bain.

intensities to others. But this is not necessarily due to differences in their algedonic tone. The very fact that one stimulus may be preferred to another when there is no conscious pleasure or pain, tends to show that the phenomenon is due to complex processes of association.[1] The pressure acting on a small area will, on this hypothesis, be judged unpleasant because we tend to think of the pain that would result if the area were much diminished, or the intensity of the stimulus much increased. In like manner, a stimulus of moderate intensity is preferable to one of very low intensity, because for low intensities perception is less distinct, and we tend, as a rule, to prefer things that we can understand. At least such appears to be the process in judgments of low dermal stimuli, so far as the writer's introspection justifies any *à priori* hypothesis.

But there are also positive as well as negative reasons for considering the phenomena of dermal pain to be most readily intelligible on the hypothesis which regards pain as a distinct sensation rather than as a *quale* or a psychic element of all states of consciousness. In the first place, pain has a peculiar quality of its own, and may occur unaccompanied by any other sensory element. When induced by haptic stimulation the consciousness of pain in the part stimulated may continue some time after the removal of the stimulus.[2]

But there are other points of difference in the time phenomena of pain and dermal sensations. If we touch a hot object the sensation of contact precedes that of pain.[3] Lehmann explains this by the difference in the reaction-times for sensations of touch and temperature.[4] The same phenomena, however, occur when the pain producing stimulus is not heat but pressure.[5] If a needle be suddenly pressed into the skin a secondary pain will appear after the sensation of pressure. In our experiments on the pain threshold for impact

[1] *Cf.* Dessoir, *op. cit.*, 186.

[2] See Chap. VII, Sec. 2.

[3] *Cf.* Dessoir, *op. cit.*, 201, 324.

[4] Lehmann, *Die Hauptgesetze des mensch. Gefühlsleben*, 44, 45.

[5] *Cf.* Goldscheider, *Physiol. Gesell.*, Oct., 1890; Die Lehre der Specif. Energien der Sinnesorgane, 1881.

stimuli the same time relation was observed.[1] Marshall argues that the pain consciousness appearing under these conditions is not necessarily a new sensation, but a sensation x in a painful phase.[2] But is this not to reduce a known state of consciousness to one that is unknown and only assumed? The same criticism, we will remark, may be made of the explanation generally given of pains arising from pathological processes in the internal organs and in the muscular and nervous tissues.

Apart from introspective and experimental evidence, the sensation theory of pain is strongly corroborated by dermal pathology. It has been known for many years that tactile anaesthesia may exist without analgesia, and analgesia without anaesthesia;[3] and, although hyperalgesia may be so acute that the slighest mechanical jar causes pain, true tactile hyperaesthesia is unknown.[4]

The different facts we have noted above certainly go to show that pain and haptic sensations are utterly disparate states of consciousness, and that in all probability there is a corresponding difference between the physiological processes. In fact, from the time when Schiff made his celebrated experiments many physiologists have believed that impulses for pain and touch pass to the brain by different paths. Goldscheider claims even to have discovered special nerves for pain; but his results have been questioned.[5] Wundt explains the physiological and pathological experiments by the altered excitability of the sensory nerves after passing through the gray matter of the cord.[6] It is possible that at least a partial cause of the delay in the appearance of pain is the development of pathological processes in the dermal tissues incited by intense stimulation. That the process of dermal pain stimulation is somewhat of this nature is made

[1] See Chap. V, Sec. 2.

[2] Marshall, *op. cit.*, 18.

[3] Wundt, *op. cit.*, I, iii; Funke, *op. cit.*, 297. Other references are given by these writers.

[4] Richet, *op. cit.*, 219.

[5] Goldscheider, *Archiv für Anat. und Physiol.*, 1885, Supp. Bd., 87. For criticism of G., *cf.* Lehmann, *op. cit.*; also Dessoir, *op. cit.*

[6] Wundt, *op. cit.*, I, 110, 437, 596; Funke, *op. cit.*, 297.

probable by an observation of Goldscheider. According to this writer the delay in the appearance of pain upon stimulation of the foot of a person afflicted with some disturbance of the circulation in that part decreased appreciably as the diseased tissues were recovering to their normal condition.[1]

That the delay is due, at least in part, to peripheral processes is also borne out by our own observations. In the course of experiments on the pain threshold for impact stimuli, the writer has observed pain in the part stimulated nearly an hour after the completion of the experiments. And again, when pain was induced only after long continued pressure, it would continue several seconds after the removal of the stimulus.[2] But whatever physiological hypothesis be accepted, there seems no doubt that there is a qualitative physical difference in function corresponding to the qualitative psychical difference in sensation.

SEC. 7. *The Quality and Intensity of Sensation.*

In the above discussion we have used the terms quality and intensity as applied to sensation. These terms have been almost universally used to denote fundamental attributes of sensation.[3] They are, however, seldom defined. The term intensity is generally used in the sense of that property of sensation which is functionally related to the intensity of the stimulus. If, as some have been led to believe, states of consciousness cannot be treated quantitatively, the term as applied to sensation clearly cannot be used in such a sense. Such a use is, however, implied in the word, since it carries with it the idea of physical quantity measurable in terms of space, to which all physical measurements are reducible. But if we reject the term altogether, applying only the predicate *qualitative* to sensational changes, in what way shall we describe subjective changes that are discontinuous, as opposed to those which are continuous? On the other hand, it may be said that, if the term quality be thus restricted, we shall have no means

[1] Goldscheider, *Deutsch. Med. Wochenschrift*, 1890, no 31.

[2] See Chap. VII, Sec. 2.

[3] *Cf.* Wundt, *op. cit.*, I, 332; Ladd, *op. cit.*, 356; Stumpf, *Tonpsychologie*, I, 350.

of distinguishing continuous sensational changes due to intensive variations from those due to non-intensive variations in the stimulus. But we do not think that there is necessarily such a difference in these modes of subjective change. The difference may be rather one of perception. The sensational change as haptic stimuli are altered in area, or as auditory stimuli are altered in pitch, is as much a continuous, and, we think, quantitative, change as is that caused by intensive variations in haptic and auditory stimuli. We are, perhaps, accustomed to think of intensive sensational differences as being measurable, rather than non-intensive differences, simply because it is a matter of familiar experience that the corresponding changes in the stimulus are quantitative changes, and we are accustomed to estimate the magnitude of the stimulus by the changes in sensation. If this view be correct, we have no term to apply universally to those changes in sensation that are continuous, as opposed to those that are discontinuous. For from the use of the term intensity in physical science it would be difficult to extend its meaning so as to cover those changes in sensation that are independent of the intensity of the stimulus.

CHAPTER III.

The Discrimination of Weights Without Effort and the Intensity of the Stimulus.

Sec. 1. *Preceding Investigations.*

The comparative ease with which the intensity of haptic stimuli can be measured renders the relation between the accuracy of discrimination and the intensity of the stimulus an attractive field for investigation. In fact, it was upon experiments with weights that E. H. Weber based his famous generalization. These experiments were, however, too few and inaccurate to base a quantitative conclusion upon them. By simultaneous pressure stimulations Weber found the least noticeable difference for 32 oz. to be 15 oz. and 10 oz., or about $\frac{1}{2}$ and $\frac{1}{3}$ of the stimulus, for the two observers. For 32 dr. the least noticeable difference for the same observers was found to be 8 dr. and 10 dr. or about $\frac{1}{3}$ of the stimulus. When the stimuli were applied in succession, the least noticeable difference was found to be $\frac{1}{29}$ to $\frac{1}{44}$ of the stimulus, but Weber does not say what intensities were used.[1]

The next research of importance is that of Dohrn, who applied the method of least noticeable difference to the investigation of the discrimination of weights of low intensities.[2] Dohrn found that for the volar surface of the right hand a weight of 1 g. had to be doubled in order for a difference to be perceived. These experiments were made on himself, and also on a boy of eleven, and must, therefore, be considered as of little quantitative value.

A series of experiments with impact stimuli conducted by Biedermann and Löwit is described by Hering, who

[1] An account of these experiments in more detail is found in G. E. Müller, *Grundlegung der Psycho-physik*, 189. Weber's original work is inaccessible to the writer.

[2] Dohrn, *Zeitschrift für Rat. Med.*, 3[te] R., X, 339.

states that they do not conform to Weber's law.[1] The method of least noticeable difference was used, and this is sufficient to discredit the results, not to speak of the absence of information as to the other details of the experiment. In experiments on lifted weights by the same experimenters, the least noticeable difference for 450 g. is stated to be $\frac{1}{6}$ g., whereas for 500 g. it is given as $\frac{1}{20}$ of the stimulus, a result that throws suspicion on the accuracy of these as well as the other experiments.

The most systematic investigation of the subject is that of Merkel, who found the least noticeable difference at 50 g. to be $\frac{1}{14}$ of the stimulus, and to be fairly constant up to 2000 g.[2] In these experiments the pressure was exerted upon the finger by the arm of a balance constructed for the purpose. The muscular reaction of the finger may, therefore, have affected the judgment. That this was the case is extremely probable, since Merkel's results closely correspond with those of the most accurate researches on lifted weights.[3] Then, too, Merkel's experiments were made on himself, and, as in all such experiments, the knowledge of the objective relations of the weights could not but have influenced the observer.

In an interesting series of experiments by Hall and Motora, the least noticeable difference was found for from 5 g. to 200 g. by changing the pressure at the rate of $\frac{4}{125}$ of the stimulus per second.[4] From 5 g. to 30 g. this quantity was about $\frac{1}{2}$ the stimulus, after which it increased considerably. In these experiments the time relations were such that the results could not be compared with those based on experiments in which successive stimulation is applied. The fact that the intensity of pressure sensations decreases rapidly, at least for low intensities, after the application of the weight, makes the problem one of considerable perplexity.[5]

We know of no other work on the subject to the record

[1] Hering, *Sitzungsber. der Wiener Acad.*, 3[te] Abth., LXXII, 342, as given in Müller, *op. cit.*, 200.

[2] Merkel, *Philosophische Studien*, V, 253.

[3] *Cf.* Fullerton and Cattell, *op. cit.*, 122; Merkel, *op. cit.*, 261.

[4] Hall and Motora, *Amer. Journ. of Psy.*, I, 72.

[5] See Chapter VII, Sec. 1.

of which we have access,[1] except that of Pierce and Jastrow.[2] In this research the probable error for 250 g. was found to be $\frac{1}{20}$ of the stimulus, and to be further decreased by practice. The relation of the probable error to the magnitude of the stimulus was not considered. In these experiments the pressure was exerted through the muscles, and therefore it is probable that, as in Merkel's experiments, the discrimination for effort is what is measured.

SEC. 2. *Further Experiments: Method of Procedure.*

There being no satisfactory determination of the accuracy of discrimination for objective, as opposed to subjective pressure, a series of experiments was made in the following way. The left hand of the observer was placed on the table, comfortably supported, and in such a manner that the palm, which was turned upward, was fairly level. The eyes of the observer being closed, two weights were placed successively upon the hand, and the observer was required to judge which was heavier by the method of right and wrong cases. When no difference was perceived the observer was required to guess. The stimuli with the smaller areas were placed on different parts of the region covered by the large area. The place was constant, however, for every two compared. The degree of confidence was recorded by having the observer use four letters, *a*, *b*, *c* and *d*, according to his confidence. The apparatus used is shown in the accompanying cut.

The weights used were cylindrical boxes, B, filled with shot, the sides being built up when necessary by stiff paper. To the bottom of the box was affixed a projecting piece of the shape of a fustrum of a cone, N, the base of which came in contact with the skin. In this way a small area of stimulation could be obtained. The material in contact with the skin was thick cardboard, so that the influence of temperature was practically excluded. Projecting upward from the centre of the box was an iron rod, AC, which, on being inserted within the glass support, M, of a chemist's stand, S,

[1] We have not access to the dissertation of Bastelberger, *Exper. Prüf. der zu Drucksinn angewandten Methoden*, Stuttgart, 1879.

[2] Pierce and Jastrow, *National Academy of Sciences*, 1884, III, 75.

prevented the weight from tipping over, as it would otherwise have done when the smaller area of stimulation was used. The weights were, of course, always placed so as to be as nearly as possible perpendicular to the hand. In order to obviate slight differences in the surface applied, the same

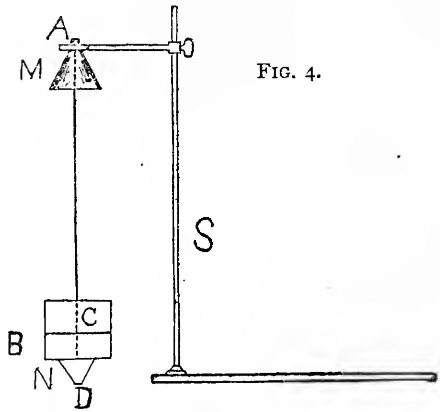


FIG. 4.

box was used to give the variable and the standard stimulus. The increment of weight consisted of a bag of shot which could be placed in the box without being noticed by the observer. The stimuli applied were as described above, except that having a weight of 3200 g. This consisted of three cylindrical kilogram weights placed one over the other. Through these weights ran an iron rod, which served as a support as with the other weights. To the base was affixed a small box loaded with shot, so as to make a total weight of 3200 g. The base consisted of circular cardboard. The time relations were fairly constant. The careful application and removal of the weights by the experimenter made it impossible to have the times of application and the intervals between the stimulations as constant as might be desired. It was found, however, by having the observer note these times, that they did not vary appreciably from 2 sec. and 3 sec. respectively. Moreover, the judgment of weight seems to be easiest as soon as the hand receives the full force of the weight, and the accuracy of discrimination does not vary appreciably when the interval between the applica-

tion of the two stimuli is not greater than 10 sec.[1] No fixed order was used in applying the stimuli, the only requirement being that for a series of 100 experiments in 50 the second weight should be heavier, and in the other 50 lighter. About 10 sec. intervened between two successive experiments, but no effort was made to have this constant. At one sitting 20 or 25 experiments in a given series were made. Then the observer rested a few minutes, and another set of experiments was made. The time devoted to the experiments at one sitting varied generally from an hour to an hour and a half. In order that the influence of fatigue might be the same for the different intensities used, the order in which the different sets of experiments for the different series was made was varied, so that a given intensity would be used as much in the first as in the latter part of the sittings. The observers were students of Psychology, with some previous practice in experimental work.[2] The experiments were begun in March, 1892, and completed in June, 1893.

With regard to possible sources of error, most of them, we think, were eliminated. In applying the weights with the hand, it is impossible to control properly the velocity of impact. The writer endeavored to obviate this difficulty by applying the weights slowly and carefully. In this way the error may be neglected for weights of sufficient intensity to cause a distinct sensation of pressure apart from one of impact. For weights of 100 g. and 200 g., however, the depression of the skin due to the pressure is so slight that impact cannot be entirely neglected. The rate of application was, however, kept as constant as practicable. Then, as is shown in Chapter V., the discrimination of weights by impact is about the same as by pressure.[3]

Another source of error lies in the slight variations of the weight from a perpendicular position and consequent pressure upon the glass support. For the purpose of inves-

[1] Cf. Weber, *op. cit.*, 545, where it is stated that there is no appreciable difference in the accuracy of discrimination after an interval of 30 sec. A far more accurate investigation of the matter is that of Fullerton and Cattell, *op. cit.*, 148.

[2] The writer would take this occasion to express his appreciation of the kindness of those who have devoted so much time to these and other experiments, and to express his gratitude for the assistance so generously given.

[3] See Ch. V., Sec. 4.

tigating this source of error, a formula was deduced for the decrease in the amount of pressure exerted upon the hand when the weight was not perpendicular. If C be the centre of gravity of the mass, D the point of application, the area being, for convenience, considered inappreciable, A the point of application of the rod upon the glass support, W the weight, and S the angular deviation from the perpendicular of the line through C, D and A; then for the loss of weight at D, which we shall call x, we shall have,

$$x = W \sin^2 \phi \frac{CD}{AD}$$

In this formula it is assumed that at the point A there is such friction that the mass is not free to move. By observing what appeared to be approximately the maximum value of ϕ in the experiments, the corresponding value of x for a weight of 500 g. was found by the formula to be 1.2 g. An experimental determination of this quality was also made by means of the balance. A 500 g. box with sharpened base was placed in the pan, so that the rod in contact with the glass support deviated from the perpendicular to about the same extent as that which was found to be the maximum deviation in the experiments. The loss of weight was found to be 1.5 g. Inasmuch as the measurement was rendered inexact by the horizontal component of the pressure exerted, the result corresponded as closely as was expected with that obtained by calculation. The true loss of weight was, however, much less than this for the stimuli used; for the place of application having considerable area, it is evident that the weight will tend to be in more stable equilibrium. Then the true error is not the average loss of weight, but the variation from this average, which is of course much less. We may, therefore, neglect this source of error entirely.

In every set of 100 experiments, in 50 of which the second weight was lighter and in 50 heavier, the percentage of right answers was calculated for both groups of 50 answers. The accuracy of discrimination, h, was determined from these data by tables based upon the well-known formula:

$$\frac{r}{n} = \frac{1}{2} + \frac{1}{\sqrt{\pi}} \int_0^{h\Delta = t} e^{-t^2} dt$$

In the tables used[1] the values of $\frac{\Delta}{\text{P.E.}}$ were given, instead of those of $h\Delta$, P.E. being the probable error, or that error which would be equal to Δ, when the percentage of right cases is 75. Tables giving the values of $h\Delta$ may be readily changed so as to give the values of $\frac{\Delta}{\text{P.E.}}$ by substituting for h the expression $\frac{.477}{\text{P.E.}}$. In some of the series it was found that the constant error, or tendency to overestimate the second stimulus, was so great that the use of a larger increment was necessary when the second weight was lighter. For otherwise the second weight would have been judged heavier the great majority of trials; and the observer, therefore, would have acquired the habit of judging the second weight the heavier, which would have vitiated the experiments. This involved the use of special formulae, which we now give. C.E. is the constant error, P.E. the probable error, T_l the value in the table for $\frac{\Delta}{\text{P.E.}}$ when the second weight is lighter, T_h the value when it is heavier, and Δ_h and Δ_l are the increments used when the second weight is heavier or lighter.

Case I.

When no constant error occurs, real or apparent, $\Delta_h = \Delta_l = \Delta$, and $T_h = T_l = T$. Then

$$\frac{\Delta}{\text{P.E.}} = \text{T},$$

whence,

$$\text{P.E.} = \frac{\Delta}{\text{T}}.$$

Case II.

When a constant error occurs, and only one increment is used, we have,

[1] Given by Fullerton and Cattell, *op. cit.*, 16. They will also be found, in different form, in *Philosoph. Studien*, IX, 145; and in Fechner, *Elemente der Psychophysik*, II[te] Auf., Leipzig, 1889, 108.

$$\frac{\Delta + \text{C.E.}}{\text{P.E.}} = T_h \text{ and } \frac{\Delta - \text{C.E.}}{\text{P.E.}} = T_l,$$

whence,

$$\text{P.E.} = \frac{2\,\Delta}{T_h + T_l}, \text{ and C.E.} = T_h \text{ P.E.} - \Delta$$

CASE III.

If a constant error occurs, and Δ_h is $> \Delta_l$ or $< \Delta_l$, we have,

$$\text{P.E.} = \frac{\Delta_h + \Delta_l}{T_h + T_l}, \text{ and C.E.} = T_h \text{ P.E.} - \Delta_h.$$

CASE IV.

If both stimuli are equal when the second is judged heavier, *i. e.*, if $\Delta_h = 0$, we have,

$$\text{P.E.} = \frac{\Delta_l}{T_h + T_l}$$

and

$$C = \text{P.E. } T_l.$$

CASE V.

If the first stimulus is greater than the second, when the second is judged heavier, Δ_h is minus, and we have,

$$\frac{- \Delta_h + \text{C.E.}}{\text{P.E.}} = T_h,$$

and

$$\frac{\Delta_l - \text{C.E.}}{\text{P.E.}} = T_l;$$

whence,

$$\text{P.E.} = \frac{\Delta_l - \Delta_h}{T_h + T_l},$$

and

$$\text{C.E.} = \text{P.E. } T_h + \Delta_h.$$

CASE VI.

If the conditions are the same as in Case V., but $\frac{r}{n} < \frac{50}{100}$ when the second weight is judged lighter, calling the value of the probability integral corresponding to $100 - \frac{r}{n}$, T_l^1, we have

$$\frac{\text{C.E.} - \Delta_h}{\text{P.E.}} = T_h,$$

and

$$\frac{\text{C.E.} - \Delta_l}{\text{P.E.}} = T_l^1;$$

whence,

$$\text{P.E.} = \frac{\Delta_l - \Delta_h}{T_h - T_l^1}$$

and

$$\text{C.E.} = \text{P.E. } T_h + \Delta_h.$$

Case VII.

If the conditions are the same as in Case VI., except that no increment is used when the second weight is judged heavier, we have

$$\text{P.E.} = \frac{\Delta_l}{T_h - T_l^1}$$

and

$$\text{C.E.} = T_h \text{ P.E.}$$

Case VIII.

If the conditions are the same as in Cases VI. and VII., except that for the second to be judged heavier, an increment is used, $+ \Delta_h$, we have

$$\text{P.E.} = \frac{\Delta_l + \Delta_h}{T_h - T_l^1},$$

and

$$\text{C.E.} = \text{P.E. } T_h - \Delta_h.$$

By the above formulæ were calculated the values of P.E. and C.E. for each set of 100 experiments.[1] That under Case II. was used in the great majority of the calculations. Such increments were generally used as would give a percentage of right cases as near as possible to 84 per cent., since fewer observations are needed for such a value of Δ in order to calculate the value of P.E.

The value of P.E. thus found is not strictly that for the standard stimulus, but is compounded of this and its value

[1] In order to test the approximate accuracy of the formulæ, the value for C.E. thus found was added to the second stimulus, and it was noted whether the conditions were such as to give about the value of $\frac{r}{n}$ as expected from the value calculated for P.E.

for the variable stimulus.[1] When the increment is very small this error may be neglected. But in some of our experiments, on account of the magnitude of the constant error, an increment was used of from $\frac{1}{5}$ to $\frac{1}{3}$ of the stimulus. This difficulty cannot be overcome unless the relation of the probable error to the stimulus is known. Inasmuch as we found that this relation was approximately that demanded by Weber's law, at least within certain limits, we corrected the values of P.E. on this assumption. The result will at least be more correct than it would if such a correction were not made, even if the probable error increased more slowly than is assumed. To make such a correction, let S be the standard stimulus, P.E. the probable error obtained from the formulæ above given, and $P.E._x$ the value of P.E. corrected for the standard stimulus. Then P.E. may be considered as approximately the arithmetic mean of the probable errors for the stimuli used. We shall have, therefore,

$$P.E. = \frac{2\,P.E._x + P.E._x\,\dfrac{(S + \Delta_h)}{S} + P.E._x\,\dfrac{(S. + \Delta_l)}{S}}{4}$$

whence,

$$P.E._x = P.E.\left(\frac{4S}{4S + \Delta_h + \Delta_l}\right).$$

SEC. 3. *Results.*

The values of P.E. given in the tables below are the corrected values. In the majority of cases they do not differ appreciably from the uncorrected values, but at times the difference is considerable. No correction was made for the constant error, since its relation to the magnitude of the stimulus is more complex.

In the appended tables the standard stimuli used are given in the first column. Then follow the different probable errors for each set of 100 experiments,[2] P_1, P_2, etc., and their averages and mean variations. The other columns give the different constant errors, C_1, C_2, etc.

[1] Cf. Müller, *op. cit.*, 21

[2] The probable errors for N. F. and J. S. are based upon 80 experiments.

Observer, N. F.; area, 8 cm.

S	P_1	P_2	P_3	P_4	Av.	M. V.	C_1	C_2	C_3	C_4	Av.	M. V.
200	31	21	16	21	22	4.	17	17	16	17	17	0.
800	255	138	72	86	137	59	120	164	176	159	155	17
1600	191	221	185	175	193	18	157	143	280	200	195	43
3200	341	474	—	—	408	44	119	222	—	—	170	57

Observer, J. S.; area, 8 cm.

S	P_1	P_2	Av.	M. V.	C_1	C_2	Av.	M. V.
800	134	97	115	18	11	30	20	9
1600	148	207	177	28	51	100	75	24
3200	244	261	252	8	149	121	135	14

Observer, R.; area, 8 cm.

S	P_1	P_2	P_3	Av.	M. V.	C_1	C_2	C_3	Av.	M. V.
100	25	27	19	23	3	17	16	37	23	9
500	102	121	83	102	13	70	120	166	119	32
1500	248	337	229	271	43	631	799	682	704	63

Observer, McW.; area, 8 cm.

S	P_1	P_2	P_3	P_4	P_5	Av.	M. V.	C_1	C_2	C_3	C_4	C_5	Av.	M. V.
100	20	24	24	14	16	19	3	1	0	15	5	2	4	4
500	33	42	31	40	33	36	4	−13	−13	2	−24	2	−9	9
1500	110	130	110	100	109	112	7	6	9	6	18	12	10	4
3200	233	183	156	196	197	193	19	96	286	200	233	285	218	56

Observer, L. S.; area, 8 cm.

S	P_1	P_2	P_3	P_4	P_5	Av.	M. V.	C_1	C_2	C_3	C_4	C_5	Av.	M. V.
100	17	20	17	19	16	18	1	−4	−3	1	6	0	0	3
500	37	29	39	42	47	39	4	−12	−2	−6	0	−11	−6	3
1500	114	100	114	103	111	108	5	0	67	13	57	69	41	28
3200	243	217	256	206	243	233	17	85	195	125	70	239	143	59

Observer, L. S.; area, $\frac{8}{64}$ cm.

S	P_1	P_2	P_3	P_4	P_5	Av.	M. V.	C_1	C_2	C_3	C_4	C_5	Av.	M. V.
100	9	14	13	16	16	13	2	1	−1	1	1	4	1	1
500	43	29	35	57	41	41	7	0	−2	−14	17	19	4	8
1500	124	91	127	105	105	110	11	59	49	111	74	74	73	15

Observer, N. F.; area, $\frac{8}{64}$ cm.

S	P_1	P_2	P_3	P_4	Av.	M. V.	C_1	C_2	C_3	C_4	Av.	M. V.
200	36	77	—	—	56	20	48	30	—	—	39	9
800	87	121	120	125	113	12	195	146	65	70	119	51
1600	227	231	196	158	203	26	259	263	215	133	217	44

Observer, J. S.; area, $\frac{8}{64}$ cm.

S	P_1	P_2	Av.	M. V.	C_1	C_2	Av.	M. V.
800	104	103	103	0	−26	−26	−26	0
1600	221	171	196	25	94	0	47	47

These results are graphically represented in the accompanying curves.

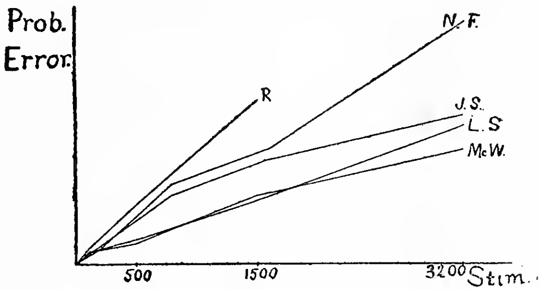


FIG. 5—LARGE AREA.

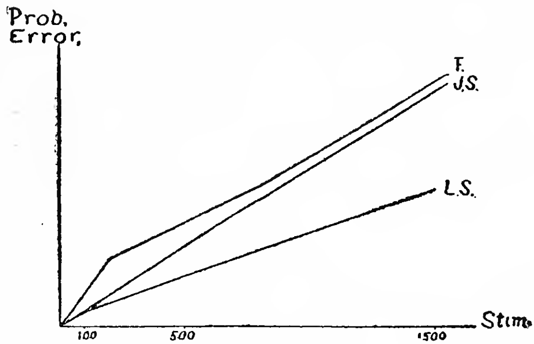


FIG. 6—SMALL AREA.

In the above experiments no stimuli were used less than 100 g. Less extended experiments were made on S. F., an excellent observer, for a standard stimulus of 5 g., the variable stimulus being 7 g. The stimuli used were cylindrical pieces of lead. To the bottom was fastened a circular piece of cardboard, having a diameter of 1.5 cm. The weights were carefully lowered upon the palm of the hand by iron rings projecting from the tops. Below are the values of the three probable errors obtained for each set of 100 experiments, and also the value of the average divided by the mean of the stimuli used.

Observer, S. F.

Stimulus.	P_1	P_2	P_3	Av. P.	$\frac{S}{P}$
5 g. and 7 g.	3.7	1.8	2.0	2.5	.4

These results may be taken as representative for good observers, since the probable error for S. F. at 1000 g., and with the same area, was found by 200 experiments to be about $\frac{1}{10}$ of the stimulus, which is fairly typical.

It is evident from the above results that Weber's law holds fairly well between the approximate limits, 300 g. and 3000 g. For very low stimuli the probable error increases much more slowly than the stimulus. For high intensities it increases somewhat more slowly, though the deviation is not very marked. It is probable that observers differ somewhat not only in their absolute accuracy of discrimination, but even in the relation of this accuracy to the magnitude of the stimulus. This is shown in the curves for L. S., J. S., and McW. (Figure 2), that of J. S. clearly departing from the straight line demanded by Weber's law. The irregular shape of N. F.'s curve is perhaps to be explained by the decided variation in his accuracy of discrimination, as shown in the tables. If it be assumed that such variation is the cause of the irregularity of the curve, it is evident that for this observer the probable error increases in direct proportion to the stimulus within the limits used.

Summarizing the quantitative results obtained, the maximum value of $\frac{\text{P.E.}}{\text{S}}$ for a set of 100 experiments was $\frac{1}{3}$,[1] the minimum $\frac{1}{20}$,[1] and the average for all observers and all intensities above 100 g. was $\frac{1}{9}$.

It is evident that individuals of about the same age and social class differ somewhat in their discrimination. Of the eight observers tested only two showed much variation from the average, N. F. and R. having as their relative probable errors $\frac{1}{5}$ and $\frac{1}{7}$. From both of these observers the writer would have expected at least as good results as from others. Both complained of a tendency to drowsiness in the course of the experiments, and to this their low accuracy may perhaps be ascribed.

[1] Calculations based upon only 100 experiments are, of course, somewhat affected by the variable error.

SEC. 4. *The Constant Error.*

It has for many years been known that in comparing two stimuli applied successively, there is in general a tendency to overestimate the second. In no instance known to the writer has a constant error of such magnitude been observed as those shown in the records of R. and N. F., which were for some stimuli as great as $\frac{1}{3}$ of the stimulus. The value of C. E. seems to increase with the stimulus, but not in direct proportion. It is very small or even negative for low intensities, but increases rapidly, apparently soon reaching a maximum. Some persons do not show any constant error except for very high intensities. Experiments on L. F. showed no constant error for 1000 g.,[1] but at 3200 g. it was appreciable.

Persons having a large constant error tend to have a large probable error. This is shown by the following average values, in round numbers, of $\frac{P.E.}{S}$ and $\frac{C.E.}{S}$ for 8 persons.

	L. F.[1]	S. F.[2]	McW.	L. S.	J. S.	W.[2]	N. F.	R.
Av. $\frac{P.E.}{S}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{9}$	$\frac{1}{8}$	$\frac{1}{7}$	$\frac{1}{5}$
Av. $\frac{C.E.}{S}$	[zero]	[zero]	$\frac{1}{33}$	$\frac{1}{33}$	$\frac{1}{33}$	$\frac{1}{16}$	$\frac{1}{10}$	$\frac{1}{4}$

Whether the constant error influences the probable error or *vice versa*, we cannot say. Possibly these magnitudes are causally related to some process affecting them both.

The constant error appears to vary more than the probable error. Below are given the relative mean variations of the probable and constant error. They are calculated by taking the mean of the values of the mean variation divided by the probable or constant error, as the case may be. We give also, for the sake of comparison, the average value of $\frac{P.E.}{S}$ for the different observers.

[1] See Chapter V, Sec. 5.

[2] The experiments on W., L. F. and S. F. will be given in Chap. V., Sec. 4, and Chap. VI., Sec. 5. The standard stimulus was not varied, being 200 g. for W. and 1000 g. for L. F. and S. F.

	N.F.	J.S.	R.	L.S.	McW.	W.
$\frac{M.V.}{P.E.}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{7}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{2}{5}$
$\frac{M.V.}{C.E.}$	$\frac{1}{5}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{3}{5}$	$\frac{3}{5}$	$\frac{2}{5}$
$\frac{P.E.}{S}$	$\frac{1}{7}$	$\frac{1}{9}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{8}$

Since there is in general greater variation in C.E. than in P.E., we conclude that these variations are to some extent true variations rather than chance variations due to the conditions of the experiment. The variation of the constant error was most noticeable in the case of an observer, E. G., on whom in two weeks over 500 experiments were made. In these experiments the value of C.E. increased so rapidly that no calculations could be made. At first it was inappreciable for 100 g., 500 g. and 1500 g.; but it increased with practice until for 100 g. it was apparently as great as the stimulus, and for the higher intensities from $\frac{1}{2}$ to $\frac{1}{3}$ as great. The theoretical importance of these variations lies in the application of the probability integral to the method of right and wrong cases, for in this integral P.E. and C.E. are assumed to be constant.

If the constant error be due to central processes, we should expect individuals having a great error for pressure to have a similarly great error for lifted weights. But this is not the case. R., who had the greatest C.E. of all the observers, failed to show the slightest trace of any overestimation in forty experiments with lifted weights. L. S. and McW. likewise had no appreciable C.E. for lifted weights of high intensity, though they had for pressure stimuli of high intensity. Not only this, but a constant error for pressure does not apparently involve one for impact. At least in 25 experiments on R., no C.E. was appreciable for 50 g. falling 20 cm.

Sec. 5. *The Confidence of the Observer.*

The confidence of an observer in estimating stimuli not differing greatly, varies from complete doubt to complete certainty. The degree of confidence depends upon the mag-

nitude of the difference of the stimuli, and consequently upon the probability of correctness.[1]

In the experiments on the discrimination of weights, observers were requested to say *a* when certain, *b* when fairly confident, *c* when less confident and almost doubtful, and *d* when unable to decide except by guessing. The results for different observers are now given. The figures indicate the percentages of times the different letters were used when the observer was right and also when he was wrong.

McW.

	a.	*b.*	*c.*	*d.*
r.	14 per cent.	44 per cent.	37 per cent.	5 per cent.
w.	4 "	29 "	55 "	12 "

L. S.

r.	1 per cent.	17 per cent.	73 per cent.	9 per cent.
w.	—	3 "	70 "	27 "

R.

r.	—	13 per cent.	78 per cent.	9 per cent.
w.	—	6 "	78 "	16 "

J. S.

r.	—	77 per cent.	92 per cent.	1 per cent.
w.	—	22 "	98 "	—

N. F.

r.	2 per cent.	24 per cent.	66 per cent.	18 per cent.
w.	8 "	27 "	59 "	6 "

As the percentage of right cases varied for different observers, we cannot express their degree of confidence by the percentage of times *a* and *b* were used. We may, however, use as a rough indication of individual differences the fraction $\frac{c}{w}$, that is, the ratio of the number of times he was confident when wrong to the total number of times he was wrong. This fraction is for L. S. only $\frac{3}{100}$; for R., $\frac{6}{100}$; for McW., $\frac{29}{100}$; for J. S., $\frac{2}{100}$; and for N. F., $\frac{27}{100}$. By com-

[1] In experiments on lifted weights described by Fullerton and Cattell, the degree of confidence varies nearly as the percentage of right cases. *Op. cit.*, 126.

paring these numbers with the relative probable errors,[1] we see that there is no relation between the two quantities.[2] As will be seen from the above results, the observers were seldom certain. It is remarkable, however, that two observers were certain 4 per cent. and 8 per cent. of the time respectively when they were wrong. We might suppose that the probability of correctness when confident would be inversely related to the degree of confidence, as shown by the fraction $\frac{c}{w}$ above mentioned. This does not appear to be the case. For McW. and J. S. the probability of correctness when confident was $\frac{9}{10}$[3]; but the values of $\frac{c}{w}$ for the two observers were quite different.

The number of times the observers were correct when guessing was greater than could be explained by chance. By taking the number of *d's* in ten separate sets, of from 100 to 150 each, and computing the percentage of right cases, it was found that in all of these ten sets this percentage was over 50 per cent., the average being 59 per cent.[4] From this it follows that to halve the number of doubtful answers and add this to the number of right cases, as has generally been done, is an illegitimate method of procedure, since based on an erroneous assumption. In the case of some observers whose confidence was small, this percentage ran as high as 65 per cent. and 70 per cent. The bearing of this on the method of least noticeable differences is, we think, quite obvious. F., about 70 per cent. of whose guesses were correct, stated explicitly that when guessing he felt no difference whatsoever, and that his judgment was entirely a guess. But apart from problems of method, such facts are of not a little theoretic importance, since they show clearly the possible accuracy of unconscious mental processes.

[1] See Sec. 4 of this chapter.

[2] Fullerton and Cattell found, contrary to this, that observers having the largest probable errors had the greatest confidence. *Op. cit.*, 126.

[3] This corresponds with the results of Fullerton and Cattell for lifted weights, $\frac{88}{100}$ being the average probability of correctness when confident for ten observers.

[4] Sixty per cent. is that given by Pierce and Jastrow, *op. cit.*; Fullerton and Cattell give 60 and 65 per cent. for two observers, *op. cit.*, 132.

CHAPTER IV.

THE PLACE OF STIMULATION.

SEC. 1. *Previous Investigations.*

In our study of the accuracy of discrimination we confined our experiments to a definite area. It has, however, been asserted on experimental grounds that the accuracy of discrimination varies for different parts of the body. We shall now turn to this aspect of the question.

The so-called tactile sensibility of different parts has generally been determined by Weber's æsthesiometer. But by this method the spatial sensibility only is measured, and we are not justified in assuming that this represents the general delicacy of the peripheral end organs. Perhaps the simplest method of testing the sensibility of different parts is to determine the threshold at these parts. The fact that the threshold is not a fixed quantity does not render this method impracticable. Aubert and Kammler[1] found by this method that there was but little difference between the different parts of the body. The face was somewhat more sensitive and the foot less sensitive than other regions, and no appreciable difference appeared between the sensitiveness of dorsal and volar surfaces. The results were quite different for parts where the hairs were shaved. Similar results were obtained by Bloch,[2] according to whom the face and palm of the hand were more sensitive than the trunk, arms and legs when shaved.

A quite different method was used by Goltz,[3] who applied to the place of stimulation the end of a rubber tube filled with water, the other end being applied to the radial artery. The stimulus was the periodic pressure from the arterial

[1] Aubert and Kammler, *op. cit.* (See Chap. II, Sec. 4.)

[2] Bloch, *op. cit.* (See Chap. II, Sec. 4.)

[3] Goltz, *Centralblatt für die Med. Wiss.*, 1863, 273.

pulsations. Goltz concluded that the sensitiveness of the skin to pressure stimuli varied in general in the same way as the discriminative sensibility for space. The method used is, however, extremely unsatisfactory. Not only was no quantitative determination made, but possible preconceptions could not but influence the process of judgment. Goltz was led to the use of such a method by observing that a branch of the temporal artery can be easily felt with the finger, but not with the hand. This apparent difference in sensitiveness is, we think, at least partly due to differences in the manner of applying the pressure. It is much more difficult to feel the arterial pulsations with the dorsal than with the volar surface of the finger, but Aubert and Kammler, as well as Bloch, found that there is no appreciable difference in the sensitiveness of the dorsal and volar regions.

A still more novel method is that of Funke,[1] who tested the sensitiveness of the skin by applying glycerine solutions of different proportions. That solution was determined the adhesiveness of which could be just distinguished from that of pure glycerine. It is clear that the accuracy of discrimination is here tested, not the threshold, and that, too, in such an inexact manner that accurate quantitative results would be impossible. Besides, the stimulus used is traction and not pressure, and as would be expected, the results are quite different from those obtained by others for pressure. Considering the inaccuracy of the method employed, Funke's results agree fairly with those of Bloch,[2] for traction stimuli, the order of sensitiveness of the principal parts of the body being: finger tips, palm of the hand, back of the hand, forearms, breast, thigh, feet and back.

Results quite different from those of the threshold investigators were found by Schwaner[3] and also by Sergi,[4] who determined the rate of vibration of a tuning fork at which

[1] Funke, *Fischer's Med. Buchhandlung*, 1891, 29, as quoted in *Zeit. für Psy.*, Vol. 2, 399.

[2] Bloch, *op. cit.*

[3] Schwaner, *Die Prüfung der Hautsensibilität*, Dissert., Marburg, 1890, as quoted in *Zeit. für Psy.*, II, 398.

[4] Sergi, *Revista di Filosofia Scientifica*, 1891, as quoted in *Zeit. für Psy.*, III, 175.

the tactile sensations began to fuse. Schwaner's results are criticised by Sergi, who points out that the amplitude of vibration, and consequently the intensity of the stimulus, is much greater for forks at low pitch. Sergi concludes that we measure the sensitiveness of the different parts of the skin by differences in the intensity of the stimulus necessary to cause a distinct sensation. It is probable that the threshold element enters into the experiment, as Sergi holds; but as the results are quite different from those of Bloch and Aubert and Kammler, it is not improbable that local differences in the duration and fusion of tactile sensations affect the results. Krohn[1] states that dermal after-images last much longer for some parts than for others.

The accuracy of discrimination for different regions was investigated by Weber[2] and also by Dohrn.[3] Weber applied weights to the forearm, and found that the increment necessary in order to be appreciated was twice as great as when the same weight was applied to the hand. Weber does not, however, mention the magnitude of the stimuli used. According to Dohrn's researches, the method of which has been described, the least noticeable difference for a stimulus of 1 g. was smallest for the thumb and fingers. Then follow the hand, forearm, breast, knee pan and feet. But, as we have already noted, these experiments are of little exact value, since not only is the method open to serious objections, but the experiments were made by the observer on himself. Then, too, according to Aubert and Kammler, individuals differ not a little in the relative sensitiveness of different parts. We cannot assume, however, even if these results are accepted, that the absolute accuracy of discrimination is measured for different places. As we have seen, the relative accuracy of discrimination is much greater for stimuli of moderate intensity; consequently, the lower the threshold for a given region, the greater would be the accuracy of discrimination at this region for low intensities.

[1] Krohn, *Journal of Mental and Nervous Diseases*, March, 1893, 11.

[2] Weber, *op. cit.*, 548. See Chap. I, Sec. 1.

[3] Dohrn, *op. cit.* See Chap. III, Sec. 1.

SEC. 2. *Further Experiments: the Accuracy of Discrimination.*

The writer made a few rough experiments on the threshold sensibility of the hand, arm and face, by the instrument already described,[1] and the results corroborated those given by Bloch, Aubert and Kammler, as well as Dohrn, assuming that the latter's results were due to the threshold differences. Experiments were also made on the discrimination of weights by the method of right and wrong cases, the probable error being determined for different parts. Six hundred experiments were made on N. F., the volar surface of the left index finger, third phalanx, being the place of stimulation. The stimuli used were 50 g., 200 g. and 800 g. The average of the six values of $\frac{\text{P.E.}}{\text{S.}}$ obtained was $\frac{13}{100}$, which was approximately the same as that obtained for the palm of the hand of the same observer.[2] Experiments were also made on L. S. The stimulus was 100 g., and the places of application were the volar surface of the left index finger and the back of the hand. The probable error for each set of 100 experiments is given below, as well as that obtained for the palm of the hand at the same time.

Observer.	P.E. for finger.	P.E. for palm of hand.	P.E. for back of the hand.
L. S.,	9.	16.	17.

On account of the comparatively small number of experiments the probable errors given are considerably affected by the variable error. Making allowance for the variable error, the results for L. S., taken together with those for N. F., indicate that apart from individual variations there is no very marked difference in the accuracy of discrimination for moderate intensities at different parts of the hand.

A further series of experiments was made on S. F. with 5 g. and 7 g. as the stimuli. The volar surface of the index finger and the hand, and the dorsal surface of the forearm, were the places of stimulation. In these experiments the error due to impact is constant for the different places, and

[1] See Chap. II, Sec. 4.

[2] See Chap. III, Sec. 3. Av. $\frac{\text{P.E.}}{\text{S.}}$ for N. F. is $\frac{1}{7}$.

does not, therefore, affect the relative results. Below are given the probable errors for the different sets of 100 experiments.

	Finger.				Hand.				Wrist.			
Stimuli.	P_1	P_2	P_3	Av.	P_1	P_2	P_3	Av.	P_1	P_2	P_3	Av.
5 g. and 7 g.	1.3	1.5	1.5	1.4	3.7	1.8	2.	2.5	9.1	3.3	2.3	4.9

These experiments show that for 5 g.–7 g. the discrimination is somewhat more accurate for the tip of the finger than for the palm of the hand, and much more so than for the back of the fore arm. The observer improved, however, greatly from practice in the experiments on the wrist. As the threshold sensitiveness is here much less, and we are not accustomed to judging stimuli thus placed, the difference was to be expected.

SEC. 3. *The Intensity of the Sensation.*

Another method of investigating the sensitiveness of different places is by comparing the intensive effect of a given stimulus with that of the stimulus applied to another region. Weber found that 5 oz. placed on the finger was judged greater than 4 oz. on the arm, but when the weights were reversed they were judged equal.[1] In order to obtain more accurate results, a weight of 5, 100, or 1000 g. was applied to the finger, and upon removal applied to the dorsal surface of the wrist, the observer being required to judge which seemed heavier. The observers were ignorant of the fact that the stimuli applied were the same. If in 10 experiments no underestimation of the stimulus was appreciable at one of the two places of stimulation, as compared to the other, we concluded that any difference in sensitiveness was too slight to be considered. When, however, the answers were such that the weight when applied to the wrist was considered much lighter, increments were added to it until it seemed equal to the standard weight applied to the finger. When 5 g. was used, increments could not be conveniently

[1] Weber, *op. cit.*

added, so 7 g. and 10 g. weights of the same area were used as comparison stimuli. Below are the results for four observers. The values of the increments given are based on five experiments.

Stimulus.	Increments added on wrist.			
	P	K	L	F
1000g - - -	0	0	0	300
100g - - -	50	0	0	90
5g - - -	>5	0	>2<5	>2<5

The above results seem to show that there is, at least for low intensities, a marked underestimation of stimuli applied to the arm, in comparison with stimuli applied to the finger. Observers differ greatly, however, that this cannot be stated as a universal law, K. not showing any appreciable underestimation. Possibly these individual differences may be due to central processes, such as unconscious allowance for sensory difference in comparing the stimuli. The fact that the underestimation tends to diminish for high intensities goes to show that different regions of the periphery do not have an intensity coefficient as Weber concluded.

Sec. 4. *The Pain Threshold.*

By the algometer already described the writer made five measurements of the pain threshold for different parts of the body. But one measurement for a given place was made at a time.. Below are the results in kilograms, with the probable errors of the averages.

Top of the head, parietal region. - - -	1.8 ±.005
Forehead, frontal region. - - - - -	1.3 ±.008
Breast, over sternum. - - - - -	2.4 ±.006
Abdomen. - - - - - - - -	1.7 ±.006
Back. - - - - - - - - -	8.0 ±.010
Right temporal region of head. - - - -	1.0 ±.003
Left " " " - - - - -	1.3 ±.004

Right thigh, ventral region. - - - - -	4.3 ±.01
Left " " " - - - - -	3.2 ±.007
Right foot, plantar surface. - - - - -	3.5 ±.009
Left " " " - - - - - -	3.4 ±.005
Right heel. " " - - - - -	7.0 ±.006
Left " " " - - - - -	5.9 ±.01
Right hand, volar surface. - - - - - -	7.3[1] ±.007
Left " " " - - - - - -	6.2 ±.007
Right hand, dorsal surface.[1] - - - -	3.3 ±.006
Left " " " - - - - -	3.6 ±.01
Right index finger, volar surface, 3d phalanx. -	3.5 ±.006
Left " " " " " "	3.3 ±.006

From this it appears that the regions over the frontal and temporal bones are most sensitive to pressure, and the heel, the back, and the muscular regions of the leg and hand the least sensitive. The sensitiveness to pain seems, then, to depend largely upon the thickness of the skin and the extent of subcutaneous tissues. The left side of the body is perhaps slightly more sensitive than the right side, but the difference, if any exists, is hardly appreciable.

[1] The measurements on the hand were carried on simultaneously with the others. But quite a number of experiments had been made on the hand before, and it seemed to have become less sensitive by about 2 k. than when it was first tested.

CHAPTER V.

Sensations of Impact.

Sec. 1. *The Threshold*[1] *for Touch.*

We should expect *a priori* that a given weight would have greater effect if applied with appreciable impact than if impact were excluded. In order to find if such were the case, a circular piece of cardboard was placed carefully upon the hand of the observer, being suspended by a delicate brass wire about 1 cm. long. The whole weighed .01g. S. F. and the writer served as observers, the one not acting as observer placing the stimulus. The observer's eyes were closed, and he did not know when the stimulus was applied. Fifty experiments were made on both observers, and the number of times they perceived the stimulus was recorded. In order to compare the results with those obtained when impact was excluded, the pressure was applied by means of the instrument described in Chap. II, Sec. 4. In order to have the area of stimulation constant, the pressure was exerted upon the card piece to which reference has just been made, the projecting wire handle having been removed. The pressure thus applied was .4 g. Below are given the percentage of times the stimuli was felt in 50 experiments.

	Impact.	Pressure.
	.01g	.4 g
F. - -	56%	66%
G. - -	52%	30%
AV. - -	54%	48%

[1] We retain this term for purposes of convenience, there being no other to denote stimuli that are perceived with difficulty.

It is evident from the above that a pressure stimulus has a much less intensive effect than one of impact, even if the velocity of the weight applied be very small. If the pressure were applied more rapidly (between 1 and 2 sec. was the time), the effect upon the dermal end organs would be more marked. But the quicker the increase of pressure the more would its effect resemble that of impact.

SEC. 2. *The Threshold of Pain.*

To find the threshold for impact stimuli a wooden upright frame was constructed 1 m. in height. A box containing a weight, of lead or brass, could slide in an open groove without appreciable friction. A scale showed the height in centimetres through which the box fell. The part of the stimulus in contact with the skin was of wood and circular in shape, the diameter being 1 cm. The box was allowed to fall by the hand, after being raised to the height desired. A wax model was made to fit the hand of the observer, so that when the hand was once placed under the movable stimulus its position could not be changed. The palm of the hand was the place of stimulation. By means of this instrument the height causing pain was found for different weights.

The experiment was conducted as follows. The required height having been previously found very roughly, the weight was allowed to fall from a height somewhat below this point. It was then allowed to fall from a height 5 cm. greater, and this was continued until pain was caused by the blow. From two to five experiments were generally made before the pain threshold was recorded. As it was found that repeated trials made the tissues more sensitive, an interval of about half a minute elapsed between the experiments.[1] Four weights were used, and with two or three exceptions but one measurement was made at one sitting for each weight. The order in which the heights for the different weights were found was the reverse in half of the experiments from that which was followed in the other half. Ten experiments for

[1] This precaution was all the more necessary because of the difference in times of the appearance of pain and the sensation of impact. This difference was occasionally very marked.

each weight were made by the writer upon himself and five upon L., an advanced student of psychology. We give below the average values in centimeters of the height necessary to cause pain for the different weights used. The probable errors of the averages are also given, being preceded by the sign ±.

Observer.	25 g.	50 g.	100 g.	300 g.
L.	32.8± .5	18.6±.4	10.6±.3	3.1±.03
G.	69.4±1.1	34.2±.3	16.8±.1	5.4± .2

If we multiply the above values for the height by the corresponding weights, we obtain the following results:[1]

	25 g.	50 g.	100 g.	300 g.
L.	820.±12.	930.±20.	1060.±30.	930.±9.
G.	1730.±27.	1710.±15.	1680.±10.	1620.±60

From this it appears that the product of the weight and the height necessary to cause pain is fairly constant. Expressing this in the form of an equation we have,

$$Wh = k,$$

which is the equation of an hyperbola. If for Wh we substitute its value, $\frac{1}{2} m v^2$, we have,

$$\frac{1}{2} m v^2 = k,$$

which expresses the relation between the mass and velocity necessary to cause pain. If we substitute $\frac{1}{m}$ for m, we have,

$$v^2 = 2k m.$$

This equation expresses the relation of the velocity and the mass, considered as factors determining the intensity of the stimulus. The equation is that of a parabola. Its meaning is that an increase of the square of the velocity has the same intensive effect on pain sensations as a corresponding increase in the mass.

[1] The probable errors of these results are found by multiplying the original probable errors by the weights.

By taking the average values of the product Wh for the two observers, and comparing these with the average values of the pressure threshold for the same area, we find that for L. a pressure of about 2300 g. is equivalent to a blow of the same mass through a height of 4 mm., and therefore a velocity of 28 cm. per sec.[1] For G., in like manner, the pressure of 4500 g. is equivalent to a blow of the same mass having a velocity 27 cm. per sec. For velocities less than these greater masses would, according to theory, be required for impact than for pressure. It is probable, therefore, that the equation does not hold for very low velocities. This we might naturally expect, since as the velocity decreases the less is the difference between impact and pressure stimulation.

SEC. 3. *The Analysis of Mass and Velocity in Impact Stimuli.*

When the same weight falls upon the skin from different heights, different sensations are aroused. In order to investigate the subjective effects of mass and velocity in haptic sensations proper, as opposed to those of pain, the writer allowed a weight of 100 g. to fall upon the palm of the observer's hand from a height of 5 cm., in the manner already described, and then found the approximate height at which a weight of 25 g., and the same area, seemed to give rise to a sensation of equal intensity. That height was considered the height required, at which, in ten or more trials, about half of the observer's judgments were 'heavier' and half 'lighter.' When the experiments were begun, the observers were asked to judge which weight seemed heavier or lighter rather than which seemed to give the more intense sensation. It was, however, evident from the statements made by the observers that they judged the intensity of the blow. Some spoke of a difference in the quality of the sensations, and two said that the weights fell with different velocities. We give below the heights determined for different observers at which the blow of 25 g. seemed equal to that of 100 g. at 5 cm.

[1] The velocity is readily calculated from the height by the formula, $h = \frac{v^2}{2g}$.

We give also the square roots of the heights, since these may be taken to represent the velocities.

Observer.	S. F.	L. F.	L.	K.	P.
h.	40	40	58	33	20
$\sqrt{h}$.	6.3	6.3	7.6	5.7	4.5

The individual differences are so great that an exact inference is impossible. It is, however, evident that in order that blows be judged of equal intensity, the height, or the square of the velocity, has in general much less effect than the weight. For if this were not the case, the average height for 25 g., to cause a blow judged equal to that from 100 g. at 5 cm. would be not far from 20 cm.[1] On the other hand the velocity appears to have a relatively greater effect than the mass. Otherwise the average values of $\sqrt{h}$ for 25 g. would be approximately 8.8 cm. or more.[2] If we assume that the above judgments are based upon equality of sensory intensity, we may conclude that to cause an intensive effect equal to that of the velocity, the mass must increase faster than the velocity, but more slowly than the square of the velocity. The great individual variations make it probable, however, that the process of judgment is somewhat complex. Possibly the fact that we are more accustomed to judge weights than velocities, may partly account for the difficulty observers have in forming a judgment. Then, too, the change in sensation due to velocity is of different modality from that due to weight. But although we cannot assume that a relation is obtained between the intensive effects of mass and velocity, this is extremely probable. The complexity of the processes of comparison is such that great individual variations are to be expected; but to whatever extent the judgments be affected by non-peripheral processes, they are doubtless based upon differences in sensory intensity.

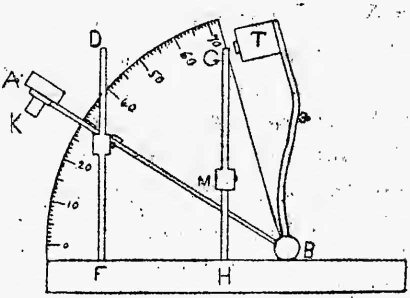
[1] 100 g. × 5 cm. = 25 g. × 20 cm.

[2] 100 g. × 2.2 cm. = 25 g. × 8.8 cm.

SEC. 4. *The Discrimination of Mass and Velocity.*

In order to investigate the accuracy of discrimination for impact stimuli, an apparatus was used constructed as shown in the cut.

FIG. 7.



An aluminium bar, AB, movable vertically, was attached to a horizontal axis, B, so that it could fall from different angular elevations. At the extremity of the bar weights of brass or lead could be attached. Metallic upright bars, attached to the wooden base, DF, GH, were provided with movable clamps, L and M. On these clamps catches were fitted by which the experimenter could let the weight and bar fall from any desired angular elevation. When a constant height was used, it was more convenient to let fall the weight from an electro-magnet, T. A scale furnished the means of adjusting the angular elevations. The stimulus was applied to the palm of the left hand, which was placed in a wax rest made to fit the hand. The hand was in contact with the cylindrical piece, K, projecting from the extremity of the weighted aluminium bar when the bar was horizontal.

Two different sets of experiments were made. In one of these the mass applied was variable and the velocity constant. In the other the mass was constant, the observer being required to estimate differences in the intensity of the blow from increments in velocity due to height. If K denote the moment of inertia of the falling mass, ω the angular velocity, W the weight, v the linear velocity, h the height

through which the centre of gravity falls, and r the distance from the axis to the centre of gravity, we have

$$\tfrac{1}{2} K \omega^2 = Wh^1$$

and

$$\tfrac{1}{2} K r^2 v^2 = Wh.$$

Since $K r^2$ is constant, when the height only is variable,

$$v = c \sqrt{h}.$$

Hence we may take $\sqrt{h}$ to represent the velocity when the height is varied. This quantity is taken as the stimulus when the mass is constant, the weight being allowed to fall successively from different heights, and the accuracy of discrimination being measured by the method of right and wrong cases. In order that the judgment might be based upon the sensation of impact only, the writer caught the lever arm of the weight by the hand the moment after it struck the observer's hand. As there was a slight rebound, it was thus comparatively easy to eliminate pressure sensations. The values of h for different values of the angular elevation of the lever arm, θ, were calculated by the formula deduced for the purpose,

$$h = r \sin (\theta + a) - l,$$

in which a represents the angle betwen the lever arm and the line passing from the axis of rotation to the centre of gravity of the lever arm and weight, C, and l represents the distance from C to the lever arm. The position of C was found by experiment. The above formula was roughly verified by measurements which, on account of the position of C, were too inexact to serve as a basis for calculations.

In the experiments the results of which are given below the standard weight was 50 g. Two standard heights were used, 5.4 cm. and 17.5 cm. The increments were for height 1.3 cm. and 3.5 cm., and for weight 10 g. The percentage of right answers varied generally between 70% and 90%. In the tables appended are given the probable errors

[1] For the deduction of this formula, the writer is indebted to Prof. R. S. Woodward, of Columbia College. If F be the impressed forces, r the lever arm, and θ the angular elevation, $K \frac{d^2 \theta}{dt^2} = Fr = Wr \cos \theta$. Hence,

$$K \int \frac{d^2 \theta}{dt^2} d\theta = Wr \sin \theta = Wh = \tfrac{1}{2} K \omega^2.$$

for the different sets of 100 experiments.[1] The second column indicates the nature of the variable stimulus, whether weight, W, or velocity, $\sqrt{h}$. When the variable stimulus is the weight, the probable error is, of course, in terms of weight. When the variable stimulus is velocity, the probable error is calculated in terms of $\sqrt{h}$.[2] In the sixth columns are given the values of $\frac{P}{S}$, the average probable error divided by the mean of the two stimuli compared. In the last columns are the values of $\frac{P}{S}$ for velocity, Rv, divided by $\frac{P}{S}$ for weight, Rw. This indicates the ratio of the accuracy of discrimination for velocity to that for weight.

$H = 5.4$ cm. $H + \Delta H = 6.7$ cm.
$\sqrt{H} = 2.32$ cm. $\sqrt{H + \Delta H} = 2.58$ cm.

Observer.	Var. S.	P_1	P_2	Av. P.	$\frac{P}{S} = R.$	$\frac{Rv}{Rw}$
S. F.	W = 50 g.	6.5	8.0	7.2	.13 = Rw.	.8
	$\sqrt{h}$ = 2.32 cm.	.33	.25	.29	.12 = Rv.	
L. F.	W = 50 g.	5.9	—	5.9	.11 = Rw.	.4
	$\sqrt{h}$ = 2.32 cm.	.12	—	.12	.05 = Rv.	

$H = 17.5$ cm. $H + \Delta H = 21.$ cm.
$\sqrt{H} = 4.19$ cm. $\sqrt{H + \Delta H} = 4.58$ cm.

Observer.	Var. S.	P_1	P_2	Av. P.	$\frac{P}{S} = R.$	$\frac{Rv}{Rw}$
S. F.	W = 50 g.	4.2	6.7	5.4	.10 = Rw.	1.0
	$\sqrt{h}$ = 4.19 cm.	.43	.43	.43	.10 = Rv.	
L. F.	W = 50 g.	5.	3.9	4.4	.08 = Rw.	.7
	$\sqrt{h}$ = 4.19 cm.	.23	.27	.25	.06 = Rv.	
L.	W = 50 g.	6.3	7.7	7.0	.13 = Rw.	1.1
	$\sqrt{h}$ = 4.19 cm.	.73	.60	.66	.15 = Rv.	

[1] See Chap. III for method of calculation. The prob. errors are not here corrected as in Chap. III.

[2] When the weight is varied there is a slight change in the velocity. For a small increment of weight, however, this may be neglected, as will be seen from the formula, $\frac{1}{2} K\omega^2 = Wh$.

In order to compare the accuracy of discrimination for blows with that for pressure stimuli, 400 experiments on S. F. and L. F. were made with a weight of 1000 g. and an area approximately the same as that used in the impact experiments. We give below the mean of the two probable errors obtained for S. F. and for L. F., divided by the mean of the stimuli compared, $\frac{P}{S}$. The values of $\frac{P}{S}$ for 50 g. are also given for comparison. These are for the greater height 17.5 cm., since, in order to have a logical basis of comparison, it is necessary to compare the relative probable errors at intensities not greatly differing. Weber's law, we have seen, holds approximately only for moderately high intensities. This is moreover evident from the table since the relative probable errors at the two heights are appreciably different.

	$\frac{P}{S}$ for pressure,	$\frac{P}{S}$ for impact.
S. F.	$\frac{1}{11}$ 1000 g.	$\frac{1}{10}$ 50 g. $\times$ 17.5 cm.
L. F.	$\frac{1}{14}$	$\frac{1}{12}$

From the results given above we may conclude, first that there is no marked difference in the accuracy of discrimination for pressure and for impact, and second, that the discrimination for velocity tends to be more accurate than that of weight. If, however, instead of calculating the probable errors for the square root of the height, we had calculated them for the height, which represents the energy of the blow, we should have found that the discrimination was better for the mass than the height. This difference in the discrimination for mass and velocity may be due to processes of perception or to actual differences in the intensive effects of mass and velocity. If we assume that the latter explanation is the true one, we can say that in order to produce equal sensory effects the relative increments of mass and velocity are related as expressed by the equation,

$$\frac{\Delta v}{v} k = \frac{\Delta m}{m}$$

in which k is a constant, having the same value as $\frac{Rv}{Rw}$ in the

tables. If this equation hold, whatever be the values of Δm and Δv, we have,

$$\frac{dv}{v} = k \frac{dm}{m}$$

whence, by integration,

$$\log. C + \log. v = k \log. m,$$

in which log. C may be taken as the constant of integration. From this we have,

$$C v = m^k$$

or,

$$v = C'm^k.$$

Substituting for k the average of the values of $\frac{Rv}{Rw}$,

$$v = C'm^{\frac{8}{10}}.$$

That is, the velocity increases as $m^{\frac{8}{10}}$. As it is more convenient to take the mass as the most direct factor in the intensive effect of the stimulus, the above relation may be expressed,

$$m = C''v^{1.3}$$

We may, therefore, write as the intensive stimulus in impact, S,

$$S = mv^{1.3}$$

The quantity k is so difficult to determine, whether or not it be variable for individuals, that the above expression is only approximate. It is, however, clear that the stimulus is to be judged a quantity varying between the momentum mv, and the kinetic energy, mv^2. In other words the mass has greater intensive effect than the energy due to the velocity, but less effect than the velocity.

It is possible that the results obtained are dependent entirely on the processes of comparison and judgment. The conclusion at which we arrived, assuming this not to be the explanation, is the same as that which we reached in the experiments on the direct comparison of the intensive effects of mass and velocity. It seems, therefore, preferable to consider the complex central process involved as causally related only to the great individual variations. It is, nevertheless, not to be assumed that these variations are entirely

of central origin. They may be due to differences in the sensitiveness of the dermal nerves to impact stimuli. The problem becomes still more complex in view of the fact that, as we have found, the pain threshold is determined, approximately at least, by the kinetic energy of the blow. This might be explained teleologically in that the injury done to the organism would tend to vary as the energy of the blow. If, as we think probable, dermal pain is a distinct sensation, with perhaps a distinct anatomical and physiological basis, it is not surprising that its stimulus should be different from that for impact sensations proper.

CHAPTER VI.

The Area of Stimulation.

Sec. 1. *The Area of Stimulation and Judgments of the Intensity of the Stimulus.*

It is a common experience that a needle or other stimulus acting on a small dermal area will cause pain, when the same pressure applied to a large area will not. The intensity of haptic sensations appears, therefore, inversely related to the area of stimulation. For the study of this problem different methods were used, the first of which will now be described.

Boxes constructed as described in Chapter III, Sec. 2, were applied successively to the volar surface of the left hand, and the observer was required to say which seemed the heavier. The area of one of the bases, which were circular, was 8 sq. cm., that of the other was .12 sq. cm. approximately, that is $\frac{1}{64}$ of the larger area. For convenience we shall speak of the larger area as A, and that of the smaller as a. Two sets of experiments by the method of right and wrong cases were carried on simultaneously. In one set an increment, which we shall call Δ_1, was added to the box A, weighted to 200 g., so that a, weighted to 200 g., was generally judged the lighter. In this set A was always the first to be applied. In the other set a different increment (Δ_2) was added to A, such that $A+\Delta_2$ was generally judged lighter than a. In this series a was always the first to be given. In this way the observer could not be influenced by the association of one area with an apparently greater intensity.

The method of calculating the overestimation of the intensity of a is as follows.

Let C.E. be the constant error due to overestimation of the second of two stimuli, P.E. the probable error, T_h and

T_l the values of the probability integral corresponding to the percentages the second stimulus is judged heavier and lighter, and N, the required constant overestimation of the stimulus acting on the smaller area. Then for the series $A+\Delta_1$ as first, and a as second stimulus, from the formula,

$$\frac{\Delta}{\text{P.E.}}=T_h,$$

we have,

$$\frac{\text{C.E.}+\text{N}-\Delta_1}{\text{P.E.}}=T_h$$

whence,

$$\text{N}=T_h\ \text{P.E.}+\Delta_1-\text{C.E.}$$

For the series a as first, and $A+\Delta_2$ as second stimulus, we have,

$$\frac{\text{N}-\text{C.E.}-\Delta_2}{\text{P.E.}}=T_l,$$

whence,

$$\text{N}=T_l\ \text{P.E}+\Delta_2+\text{C.E.}$$

The values of P.E. and C.E. having been found by experiments carried on simultaneously with these, the values of N were calculated from the above equations. With W., an advanced student of psychology, 400 experiments were made, and the values of N for each set of 100 were as follows:

Observer, W.	N_1	N_2	N_3	N_4	Av.N.	$\frac{N}{S}$
Stimulus, 200 g.,	65	68	75	69	69	1/3

The overestimation of the weight applied to about $\frac{1}{64}$ of the larger area was, therefore, approximately $\frac{1}{3}$ of the stimulus. Experiments were also made by another method, and on a number of observers. The method of right and wrong cases was used, but the relations of the stimuli were different. The first stimulus was constant, and had the smaller area. The second stimulus was part of the time $A+\Delta_1$, and part of the time $A+\Delta_2$, the values of Δ_1 and Δ_2 being such that $A+\Delta_1$ was judged heavier and $A+\Delta_2$ lighter than a, the constant first stimulus. By this method from twenty to fifty experiments were made on W., N. F., L. S., and McW., all being subjects in the experiments on discrimination of weights already described. Not enough experiments were

made to base a calculation upon them, but enough to estimate roughly the overestimation. The results for W. were corroborative of those already obtained, both for 100 g. and 200 g. L. S. and McW. showed an overestimation at 500 g. of about $\frac{1}{3}$ the stimulus, closely corresponding to that of W. N. F. showed, however, an appreciable tendency to *underestimate* the weight of *a* at 100 g. and also at 200 g.

By a third method the increment added to a weight with area *A* pressing on the hand in order to make it appear equal to the same weight lifted was compared to the increment which had to be added when the area was *a* instead of *A*. Below are the values of the increments obtained for a 200 g. weight, each based upon five or more experiments. The observers were, of course, ignorant of the purpose of the experiment, as well as of the magnitude of the increments.

	P.	K.	L.F.	S.F.
A.	37	0	150	148
a.	0	0	0	140

In only two of the four observers does this overestimation due to the area appear very marked in these experiments. From this, and from the fact that by a different method there was not only no overestimation found for N. F., but even the reverse, we may conclude that this constant tendency is by no means universal.

The fact that individual variations are such that we cannot infer a relation between the area and the intensity of stimulation does not prove that such a relation does not exist. An observer may unconsciously allow for this overestimation in his judgment, though this was certainly not the case with N. F., since this observer supposed the larger area would seem heavier. It is difficult, moreover, to explain in this way the results obtained by the last method. But in the direct comparison of weights of different areas, great difficulty is experienced by some observers in forming a judgment. The sensations seem heterogeneous and therefore incomparable. It is only by abstraction that a judgment of intensity is possible; and in this process it is but natural that individuals should differ greatly.

SEC. 2. *The Tactile Threshold.*

If the intensity of haptic sensations is related to the area of stimulation, we should expect the tactile threshold to vary with the area. For the purpose of investigating this problem, the writer cut circular pieces of card board about 3.5 mm. and 10.7 mm. in diameter, their areas being, therefore, approximately in the ratio 1 : 9. The weights of the cards were about .01 g. and .05 g.; but this may be neglected, since only at the moment of application of the weight do pressure stimuli of low intensity have any sensory effect. When one or both of these cards had been placed on the palm of the hand of the observer, who was blindfolded, the pressure necessary to affect consciousness was found by the apparatus and methods described in Chapter II. The same corrections are also made. Ten experiments for each area were made on L. F. by the writer, and as many on the writer by S.F., who was carefully instructed as to the precautions necessary. A third set of experiments was made in which no card was used, the pressure being exerted directly upon the skin by the vertically projecting bristle of the instrument. The diameter of the bristle being about .4 mm. the area applied was about 1.2 mm. Below are the results in grams for each group of ten experiments.

	S. F.			H. G.		
Area.	Av.	Max.	Min.	Av.	Max.	Min.
1 mm.	.2	.5	.1	.5	1.	.2
10 mm.	.9	1.7	.3	1.4	3.2	.3
90 mm.	1.9	2.7	1.	1.6	2.5	.4

It appears from this that the tactile threshold varies with the area of stimulation, but that it increases much more slowly than in direct proportion. From what we have already said regarding the possibility of regarding the threshold as a definite quantity more exact results could hardly be expected. Strictly speaking we are not justified in using the term threshold, but it is convenient to do so, if we bear in mind that no absolute quantity is measured, but only the relative sensory effect of stimuli acting on different areas.

SEC. 3. *The Threshold of Pain.*

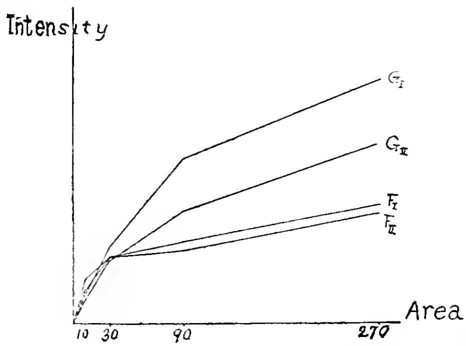
In order to investigate this relation with accuracy, wooden cones of hard wood were cut across vertically by a lathe at three points so determined by calculation that the diameters of the sections would be approximately 6.18 mm., 10.70 mm., and 18.54 mm. A still smaller circular base, about 3.56 mm. in diameter, was made by rotating a wooden cone cut by hand over sand paper until the diameter required was obtained. In this way areas were obtained of approximately 10, 30, 90, and 270 sq. mm. The diameters of the two smaller bases were measured a number of times on the dividing engine. The averages of five readings were 6.21 cm. and 3.58 cm., which shows that the areas are sufficiently accurate. In applying the pressure the algometer already described was used. The pressure was exerted upon the desired area by fitting the upper part of the wooden piece, the base of which had the area in question, into another wooden piece. Into this in turn could be fitted the projecting cap of the algometer. The rate of increase of the pressure was kept as constant as possible, and this was as great as was consistent with taking the readings accurately. The error due to the increase of pressure between the appearance of the pain and the taking of the reading is corrected as in Chapter II. It will, therefore, not affect the results. The place of stimulation was the palm of the hand. With F. the right hand was used, but the left hand was used in experiments on G. But four experiments were made on one day, one for each area. In half of the experiments the order in which the different areas were used was the reverse of that which was followed in the other half. Though the experiments made by the writer on himself were purposely extended over several weeks, a gradual decrease of sensitiveness to dermal pain was observed. This is not so noticeable in the results found for S. F. The averages, with their probable errors,[1] are given below, for each set of five experiments. The figures indicate kilograms.

[1] As the quantity determined increases appreciably for G., the use of the probable error is not justified, and it is not given.

Observer.	Group.	Area.			
		.1 cm.	.3 cm.	.9 cm.	2.7 cm.
S. F.	1st five.	1.4±.0	2.7.1±	3. ±.1	4.5±.2
	2nd five.	1.9±.1	2.7.±1	3.4±.1	4.8±.2
G.	1st five.	1.	2.6	4.6	7.3
	2nd five.	1.3	3.1	6.8	10.
Av.		1.4	2.8	4.4	6.6

These results are represented graphically in the accompanying curves.

FIGURE 8.



As the curves obtained differ somewhat, it is impossible to express the relations by a simple expression. They approximately are logarithmic curves, but for the largest area the increase of intensity is too great. As the stimuli are in geometrical progression, the logarithmic relation requires an arithmetical increase of the area. We may, therefore, test the results by finding the differences between the threshold at the different areas. These differences are as follows:

	10 and 30 mm.	30 and 90 mm.	90 and 270 mm.
S. F.	1.3	.3	1.5
S. F.	.8	.7	1.4
G.	1.6	2.0	2.7
G.	1.8	3.7	3.2
Av.	1.3	1.7	2.2

The increments appear to increase with the area, whereas an arithmetical progression requires that they be constant.

But as will be seen by inspection of the above figures, the increase is within the limit of individual and other variations.

SEC. 4. *Theoretic Interpretation of Experiments on the Intensive Effect of the Area.*

We have by three entirely different methods investigated the intensive effect of the area of stimulation. By each one of these methods we have arrived at the same result, that the intensity of pressure sensations is inversely related to the area of stimulation. We have seen that the intensity causing pain is approximately proportional to the logarithm of the area. Hence, as the intensity of the stimulus increases, its effect is the same as that from the decrease of the corresponding logarithm of the area. The experiments on the tactile threshold and on judgments of intensity, though not admitting of such an interpretation, nevertheless seem to show that the intensity threshold increases much more slowly than the area. Hence we may write as an approximate expression of the relation between the intensity and the area of the stimulus which must exist in order that equal subjective intensive effects be produced,

$$I = \frac{K}{\log A}$$

If we could assume Fechner's law we could substitute the value of I in the equation,

$$S = K_1 \log I,$$

in which S denotes intensity of sensation, and obtain,

$$S = K_1 \log \left(\frac{K}{\log A}\right).$$

That is, the intensity of the sensation increases as the logarithm of the reciprocal of the logarithm of the area multiplied by a constant. If, as Hering and some others hold, S is directly related to I, the relation between S and A would be an inverse logarithmic one. If, as the majority of psycho-physicists believe, S increases much more slowly than I, even though not in logarithmic proportion, it would increase correspondingly more slowly than the reciprocal of the logarithm of the area. The inverse relation here exist-

ing is contrary to what we might expect by the analogy of other senses. In the case of temperature sensations Weber showed that the intensity increased with the area.[1] Müller-Lyer found that the least noticeable difference for visual stimuli increased with the area of stimulation in about the same proportion as when the intensity was variable and the area constant.[2] With pressure stimuli the conditions are apparently the same; the larger the area the greater the number of nerve fibres stimulated.[3] If the stimulus were the physical pressure exerted, the intensity would, we think, increase with the area. For not only are more nerves acted on by larger areas, but the pressure increases in direct proportion to the area, provided the force applied be constant. As the exact reverse effect is produced upon the sensory end organs, the stimulus can not be considered mere pressure, but rather as work done upon the skin and subcutaneous tissues.

In sensations of impact the stimulus is, as we have seen, the energy of the moving mass or a quantity more approaching to the momentum.[4] In the case of impact stimuli, therefore, we should not expect the intensity of the sensation to increase, but rather to diminish with the area. For the greater the area the less the energy or momentum transferred to the dermal tissues within a given area. When no impact, but only pressure, is exerted, then the stimulus is the work expended in overcoming the resistance of the skin. The work done is independent of the area of stimulation, as it depends upon the impressed forces. Consequently, the greater the area the less is the work done at any one point in the region affected by the pressure; in other words, the less the intensity of stimulation. If, moreover, the function of the touch corpuscles be protective, as some suppose, the stimulus will meet with greater resistance, the greater the area upon which it acts.

[1] Weber, *op. cit.*, 553; also Dessoir, *op. cit.*, 297.

[2] Müller-Lyer, Archiv für Anat. und Physiol., 1889, Supp. Bd.

[3] Funke even states that the intensity increases with the area. *Op. cit.*, 331.

[4] The movement theory of haptic stimulation was, we believe, first advanced by Lotze, *Med. Psychologie*, 198.

This leads us to the consideration of the peculiar phenomena of dermal pressure from liquid or gaseous bodies. The atmosphere exerts a pressure upon the body of 1.03 k. per sq. centimeter, but has no effect upon consciousness. When the hand is placed in a fluid, even of considerable density, no pressure is felt except at the surface. These phenomena are intelligible, when we consider that the process of haptic stimulation involves a transference of energy. The element of time is of not a little importance, but will be considered later. The ring effect observed when the hand is plunged in mercury has its counterpart in the relatively great intensity of pressure sensations from solid stimuli in the region of the perimeter of the surface applied. A kilogram weight placed on the hand will be felt most distinctly at the edge. That the skin is more affected in this region is shown by the dark red line from vaso-motor disturbance that here appears when the stimulus is removed. It is not necessary, therefore, to explain the phenomena by such hypotheses as that of Meissner, who held that the process of pressure stimulation was an oscillatory action in the tactile corpuscles.[1]

SEC. 5. *The Area of Stimulation and the Discrimination of Intensity.*

In the chapter on the accuracy of discrimination for different intensities, two areas were used, 8 cm. and $\frac{8}{64}$ cm., approximately. In addition to the experiments there described, 1000 experiments were made on W. In 500 of these the larger area was used, and in the other 500 the smaller area. In the following table will be found the probable and constant errors based upon each set of 100 experiments.

Stimulus, 200 g.

Area.	P_1	P_2	P_3	P_4	P_5	Av. P.	C_1	C_2	C_3	C_4	C_5	Av.C.
8 cm.	37	19.	22.	20.	14.	22.	15	10	15	9	5	11
$\frac{8}{64}$ cm.	33	56	10.	14	16	25	32	8	9	3	12	13

[1] Meissner, *Zeit. für Rat. Med.*, 3[te] R., VII. *Cf.* Funke, *op. cit.*, 328, for a criticism of Meissner's theory.

From these results it is evident that the accuracy of discrimination of this observer was not on the whole appreciably altered by the variation of the area. The same might be said of the constant error. The variation of the probable error for the smaller area is, however, so great that in spite of the large number of experiments, the results do not admit of an exact interpretation. By comparing the average values of $\frac{P}{S}$ for the observers S. F., J. S. and L. S., on whom experiments were made for both areas, we obtain somewhat more satisfactory results. By referring to the tables, Chap. III, Sec. 3, we see that although N. F.'s probable errors are very variable, especially for the smaller area, those of L. S. for both areas are fairly constant. The following table gives the average values of $\frac{P}{S}$, the probable error divided by the stimulus, for all intensities used.

Area.	Average values of $\frac{P}{S}$.			
	N. F.	J. S.	L. S.	W.
8 cm. . .	.13	.11	.10	.11
$\frac{8}{64}$ cm. . .	.18	.12	.10	.13

Although W. and N. F. appear to judge stimuli of the larger area more accurately, there is no appreciable difference for J. S. and L. S., the most constant of the four observers.

Sec. 6. *The Intensity of Stimulation and the Discrimination of Areas.*

It was not our purpose to enter into a discussion of the problem of tactile space perception. But having found that the discrimination of intensity was not uniformly affected by the area of stimulation, the question suggested itself whether the converse was true. For the purpose of finding if this were so, weights were used of 200 g. and 800 g. Two

standard areas were used, being approximately 32 mm. and 11.3 mm. in diameter, and therefore 800 mm. and 100 mm. in area. The bases of the boxes applied were covered with stiff paper cut so as to be circular in shape. We were not investigating the absolute accuracy of discrimination of areas, nor yet the relation of this to the magnitude of the area; consequently any errors due to the method of obtaining the different areas may be neglected. The probable and constant errors given in the tables are, of course, in millimeters. They were obtained, as in other experiments, by the method of right and wrong cases, from the percentage of right answers in a hundred experiments. In these experiments the area is considered the stimulus, S, and the increment Δ, is the difference between the area of the standard, 100 or 800 mm., and that of the area to be compared. The magnitude of the increment is obtained by the equation,

$$\Delta = \pi\,(r^2 - r_1^2,) = \pi\,(r + r_1,)\,(r - r_1,),$$

in which r and r_1 are the radii of the bases. The first of the two tables gives the results for three observers, the smaller area being used. The second table gives corresponding results for the larger area. But 100 experiments were made upon N. F. and W. for each of the probable errors calculated. The probable errors for L. S., however, are each calculated from the results of 300 experiments.

Area, 100 mm.

	Probable Error.			
Weight.	L. S.	N. F.	W.	Av.
200 g.	43	40	21	31
800 g.	31[1]	—	47	39

Area, 800 mm.

200 g.	73	47	41	53
800 g.	91[1]	70	69	73

[1] A weight of 1000 g., instead of 800 g., was used in experiments on L. S.

The figures above given show that the discrimination for areas is not as accurate for the high intensity, the probable errors for 800 g. being in four cases out of five much greater than for 200 g. The one exception appears in the experiments on L. S. for the smaller area. In conducting these experiments, however, it was found that the accuracy of discrimination for 800 g. increased to such an extent from practice that the variable area had to be changed in order that the increment might not be too great for accurate calculation of the probable error. In the first few experiments made, then, the accuracy of discrimination was undoubtedly much less for 800 g. than for 200 g. There was, however, no appreciable improvement from practice after thirty or forty experiments. The apparent exception, therefore, partially confirms the results obtained for the other observers.

If we compare the second table with the first, we note that the probable error for the larger area, although appreciably greater, does not increase in proportion to the areas. It is possible that in these experiments what is really discriminated is the linear relation, that is, the relative diameters of the circles. This is, however, not probable; for the difference felt seems to be a qualitative difference in the sensations from which that of space is inferred. The change in sensation corresponding to change in the area of stimulation might, indeed, be supposed to be merely a quantitative change in extensity. But the great difficulty observers have of comparing intensities of different areas confirms the results of the writer's introspection, that this change in sensation is not primarily a spatial change.

CHAPTER VII.

The Time of Stimulation.

Sec. 1. *The Intensity of Haptic Sensations in Relation to the Time: Low Intensities.*

The intensity of visual and temperature sensations is clearly related to the time of stimulation. But if a weight of low intensity, and of not too small an area, be applied to the skin, the resulting sensation will continue but a few seconds, and will not increase, as might be expected, with the time of application. It is largely on this principle that the phenomena of gaseous and liquid pressure may be explained. The pressure of the atmosphere is fairly constant, and is, therefore, not perceived. Meissner observed that melted wax allowed to harden on the hand had no sensory effect, although his explanation is different from that here given. When the hand is plunged in mercury and held in the same position, the pressure remains constant, and it is only when the hand is moved that the pressure sensation is distinct.[1] Hall and Motora found, contrary to what we might expect, that the discrimination of gradual pressure change was best for the slowest rate of change of stimulus that was used, $\frac{1}{250}$ of the stimulus per second.[2] In these experiments the observer had first to decide whether the stimulus was increasing or decreasing; and it is probable that, as suggested by the writers, the cause of the decrease in the accuracy of discrimination was the distracting effect of sudden changes upon the attention, and consequently upon the accuracy of perception.

We intended to make experiments on judgments of intensity in relation to the time of stimulation, but the appa-

[1] *Cf.* Meissner, *op. cit.*

[2] Hall and Motora, *Am. Journ. Psy.*, I, 87.

rent impossibility of eliminating the error arising from memory of the standard stimulus led us to confine ourselves to qualitative observations. We did, however, make experiments on the intensive effect of the time for stimuli of such low intensity as to be perceived with difficulty. The instrument used was that already described, by which pressure was exerted by the hand of the experimenter.[1] There was no means of regulating the rate of application of the pressure except the judgment of the experimenter, and the results must therefore be considered as inexact. The experimenter practised himself in increasing the pressure at such a rate that it took 8–10 sec. to reach a pressure of .4 g. In like manner a time of 1–2 sec. was obtained. The third time of application was the shortest, $\frac{1}{4}$–$\frac{1}{6}$ sec., the increase being as rapid as was consistent with accuracy. The rates of increase were, therefore, .05–.04 g., .4–.2 g. and 2.3–1.6 g. per sec. The maximum pressure, .4 g., was, of course, kept fairly constant. Fifty experiments were made for each rate by the writer on S. F., and as many by S. F. on the writer. In the tables below are the percentages of times the stimulus .4 g. was perceived at the different rates of increase.

	.05—.04 g. per sec.	.4—.2 g. per sec.	2.3—1.6 g. per sec.
S. F. - -	10%	34%	82%
G. - -	2%	30%	82%
Av. - -	6%	32%	82%

From these results it is evident that the sensory effect of pressure stimuli increases with the rate of application. This is what we should expect on the assumption that the intensity of pressure sensations decreases rapidly with the time of application.

As for the theoretic interpretation of these results, we judge them corroborative of the movement theory of dermal stimulation. The stimulus is not to be considered mere pressure, but the energy expended upon the dermal tissues. From this point of view we are not justified in identifying the time of application of a pressure stimulus with the time

[1] See Chap. II, Sec. 3.

of actual stimulation. If a vibrating tuning-fork of sufficient amplitude to cause a distinct sensation be placed in contact with the skin, there is continued intermittent stimulation, and the sensation does not decrease as when a pressure stimulus is applied. The intermittent process of stimulation by the tuning-fork is similar to that of visual and auditory stimulation, for the physical stimuli are successive transformations of energy.

The time phenomena of pressure stimulation may also be brought under the general law that it is not static conditions, but changes in the environment that give rise to the 'nervous shock' of Spencerian psychology. It is such conditions of the environment that the organism needs to perceive in order to coördinate its motor activities for purposes of self-preservation and reproduction. The cataplectic shock so frequently experienced when one is unexpectedly addressed, shows how sensitive the central nervous system is to sudden peripheral changes. But we need not depend only upon observation for proofs of this principle. It is well known that motion on the skin may be perceived within the circumference of Weber's sensor circles; and Hall and Donaldson found that the perception of motion is independent of direction, and is clearest immediately after the motion begins.[1] In fact, the time phenomena of dermal sensations point clearly to the difference theory of sensation, according to which change in the objective environment and in the subjective mind is the *sine qua non* of sensation.[2] The psychological generalization has, moreover, a demonstrable physiological basis. Only on making or breaking an electric circuit is a motor nerve stimulated. More closely bearing on our problem are the experiments of Fontana, who found that pressure could be applied so gradually as to kill a motor nerve without inducing muscular contraction. Similar results have been obtained for temperature stimuli.[3]

[1] Hall and Donaldson, *Mind*, X, 556.

[2] Cf. Höffding, *op. cit.*, 138, 141; Dessoir, *op. cit.*, 188.

[3] Heinzmann, *Archiv. für gesammte Physiol.*, VI, 222.

SEC. 2. *High Intensities.*

When stimuli beyond a certain intensity are applied, the phenomena are quite different. If a kilogram weight be placed on the hand, the intensity of the sensation seems gradually to increase and to pass into pain. Müller observed that a sensation of pricking and of having a limb 'asleep' could be caused by long-continued pressure stimulation.[1] Similar observations on temperature stimuli were made by Weber. Thus it appears that heterogeneous sensations may be caused by variations in the time of application of dermal stimuli.

For the purpose of investigating the relation between the time of pressure causing pain and the intensive pain threshold, weights of different magnitudes were placed in a balance pan so as to act on a constant area of the palm of the hand, and the time was noted which elapsed before the appearance of pain. To the under side of the balance pan a wooden piece was fastened, the circular end of which, 1.5 mm. in diameter, came in contact with the palm of the hand. The longer times were recorded by a watch, and the shorter times by the Hipp chronoscope, a Morse key being so placed that the observer could close the circuit as he applied the weight without appreciable error. The place of stimulation was varied within a radius of about 1 cm.; otherwise there would have been danger of alterations in the condition of the skin at the place of stimulation as the experiments progressed. Ten experiments were made for each weight on two observers, S. F. and the writer.[2] But one experiment for a given weight was made on any one day, and not more than three or four times a week were these experiments made. The order of half of the experiments was the reverse of that of the other half. The figures below indicate the time in seconds for the various stimuli to cause pain. Only the averages are given, together with their probable errors.

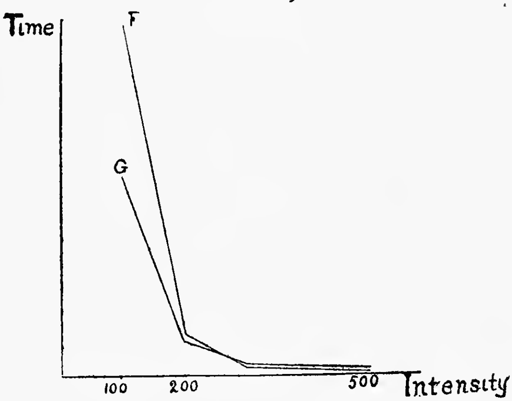
[1] Quoted by Weber, *op. cit.*

[2] Experiments were begun on another observer, but were not completed. It was evident, however, that results would have been obtained similar to those obtained for G. and S. F.

Observer.	100 g.	200 g.	300 g.	500 g.
S. F. . .	294 ± 30.	37 ± 3.6	8. ± .9	3.3 ± .2
G. . . .	167 ± 17.	34 ± 2.7	12 ± 1.5	6. ± .06

The results shown above are represented graphically in the accompanying curves.

FIGURE 9.



The experiments prove beyond question that the pain threshold is functionally related to the time as well as the intensity of stimulation. The results obtained are not sufficiently exact to admit of an analytical expression. It appears that the time curve approaches O as its limit for high intensities, and that for low intensities it either increases indefinitely or approaches as its limit an asympotic line parallel to the axis T. That the ascending branch does approach a limit is evident from introspective observation. Weights of low intensities soon cease to be perceptible, and never become painful. Thus the clothing we wear exerts continual pressure, but is never painful. The curves obtained may be said, therefore, to resemble rectangular hyperbolas. If the relation between the time and intensity causing pain could be thus represented, it would be expressed analytically by the equation,

$$(I-h)\ T = k,$$

in which h is a constant, being the intensity below which stimuli are never painful. To obtain the relation between the time and intensity having equal intensive effects as the sensation increases in intensity, we can substitute in the above equation the reciprocal of T, since the intensive effect would of course decrease as the threshold increases. We then have,

$$T = \frac{I}{k} - \frac{h}{k}.$$

From this equation, that of a straight line, we see that as regards their intensive effect on pain sensations, the intensity and time of the stimulus increase in direct proportion.

A striking feature of the above experiments is the great variation in the time before the pain appears. This is, however, only partially a true variation. It is generally difficult to decide, especially for low intensities, when the stimulus becomes painful. This is quite the contrary of what we found to be the case when the intensive threshold was being determined. There is, however, no constancy even in the manner of variation. At times for the 100 g. stimulus the pain would come very suddenly, only to cease and reappear, but generally the appearance of pain was very gradual. The sensation seemed to increase in intensity before the pain was distinctly felt. In the experiments on 100 g. a latent period seemed to elapse before any appreciable increase in intensity began. This latent period for S. F. seemed to be on the average about half of the total time. The two observers disagreed as to whether there was any appreciable decrease in intensity before the increase began.

We are not justified in assuming that the relation between the time of pressure and the intensity of pain holds also for pressure sensations. The gradual appearance of the dermal pain, preceded by an apparent increase of pressure intensity, undoubtedly points to such a conclusion. But the results admit of another interpretation. It is quite possible that the mind confuses the incipient pain with the pressure sensation. As we know dermal pain to be related to the intensity of the stimulus, and as we are not accus-

tomed to think so much of the time as a factor, it is but natural that we should judge the change in sensation to be due to more intense stimulation. We should then consider this change an intensive change, until the painful element became so clear as to be distinct in consciousness. That it *is* difficult to distinguish heterogeneous sensations of very low intensity is shown by the experiments of Wunderli.[1]

Moreover, our experiments are, we think, corroborative of the theory that sensations of pain and pressure are utterly disparate. For, otherwise, how could we explain the fact that the intensity of pain increases with the time, whereas the intensity of pressure sensations, at least for low intensities, decreases rapidly with the time? Equally difficult to account for on the algedonic tone theory is the continuation of the pain after the cessation of stimulation, which was very marked for 100 g.

But if pain be a distinct sensation, with a distinct physical basis, the results are quite intelligible. From this point of view, we can understand how pressure without appreciable transformation of energy can be considered a stimulus to pain. Unlike haptic sensations proper, dermal pain furnishes the sensory data for perceptions, not of the objective environment, but of the subjective self; and may, therefore, be induced by any stimulus of sufficient intensity. This view finds further support in teleological considerations. If pain exist for the purpose of warning the higher centres of injury done to the tissues, the intensity of pain would, we should expect, be related to the time of pressure; for the longer the time of pressure the greater the resultant injury to the tissues.

[1] See Ch. I, Sec. 2.

SUMMARY.

We shall now present a brief summary of the more important results of the investigations described in the preceding pages.

A.—EXPERIMENTAL.

1. Hot and cold stimuli are overestimated for low intensities, but not for high intensities.
2. The estimate of the intensity of haptic stimuli increases for low intensities much more slowly than the stimulus; but as the stimulus approaches the pain threshold, the estimate of intensity increases much faster.
3. The intensive pain threshold, for pressure acting on .5 cm. of the hand, varies greatly with individuals, the average being 5400 g. Age and sex appear to have less effect than individual differences.
4. The average value of the least perceptible pressure acting on an area of .9 cm., the rate of increase being about .3 g. per sec., was 1.9 g. for S. F. and 2.6 g. for G. The intensive range of haptic sensations for these observers, as based upon these measurements, was about 1700.
5. Weber's law holds approximately for weights greater than 100 to 500 g. For low intensities the probable error increases much more slowly than the stimulus.
6. The average value of the ratio of the probable error to the stimulus for stimuli of from 100 g. to 3000 g. is $\frac{1}{9}$.

7. The constant error is frequently very great for pressure stimuli. It increases with the stimulus, but the relation is complex, and is subject to great individual variations. Some observers have no constant error except for stimuli of very great intensity. The constant error is more variable than the probable error. Its magnitude seems inversely related to the accuracy of discrimination. A great constant error for pressure does not necessitate one for lifted weights.
8. The degree of confidence in the perception of intensive differences varies greatly for individuals, the proportion of wrong judgments of which observers were confident ranging from $\frac{1}{3}$ to $\frac{1}{50}$. The probability of correctness when confident was for most observers from .8 to .9. There is no relation between either of these quantities and the accuracy of discrimination. The percentage of correct guesses varied from 52% to 70%, the average being 59%.
9. The accuracy of discrimination for weights of 100 g. or more is, on the average, not appreciably different for the palm of the hand, the back of the hand and the volar surface of the index finger. For 5–7 g. the accuracy of discrimination, as found from one observer, for the palm of the hand and the back of the forearm, is less than for the index finger, but improves greatly by practice.
10. Stimuli of low intensity placed on the forearm, are judged lighter than when placed on the palm of the hand or the index finger.
11. The pain threshold for pressure varies with the place of stimulation, being greatest where the skin is thick and separated from the bone by muscular tissues. The temporal region of the head is the most sensitive, and the palm of the hand, the thigh, and the heel, are among the least sensitive parts.

12. Weights of .01 g. are about as easily perceived when impact is not entirely excluded, as weights of .4 g. when pressure only is applied, the time of application being 1–2 sec.
13. The pain threshold for impact stimuli is determined by the product of the mass and the square of the velocity.
14. In judgments of the intensity of impact stimuli the mass has in general more effect than the square of the velocity, but less than the velocity.
15. Differences in velocity are perceived, on the whole, more accurately than differences in mass, but much less accurately than differences in the square of velocity. Individuals differ greatly, however.
16. The discrimination for moving weights is about the same as for weights applied without appreciable impact.
17. The area of stimulation does not, on the whole, affect the accuracy of discrimination for weights. But individual peculiarities appear in the results obtained.
18. Pressure stimuli of small area are generally overestimated. The extent of overestimation of intensity for an area $\frac{1}{64}$ of 8 cm. was on the average $\frac{1}{3}$.
19. The probability that a stimulus of very low intensity will be perceived is inversely related to the area of stimulation.
20. The pain threshold increases with the area of stimulation in approximately a logarithmic proportion,
21. The discrimination of areas is much better for stimuli of 200 g. than for stimuli of 800 g.
22. The relative accuracy of discrimination for areas is not constant, but is greater for large areas.
23. The probability that pressure stimuli of very low intensity will be perceived increases with the rate of increase of the stimulus.
24. The relation between the time and intensity threshold of pain is approximately expressed by an hyper-

bolic curve. The appearance of pain as the time of stimulation is increased, is generally very gradual and difficult to determine. There is an intensive limit below which stimuli never cause pain.

B.—Theoretical.

1. There is no basis for the alleged identity of haptic and temperature sensations.
2. Pain, tickle and pressure sensations are heterogeneous sensations induced by quantitative changes in the intensity of the stimulus. Dermal pain itself is probably a sensation and not merely an intensive form of the algedonic tone.
3. Touch and pressure sensations are qualitatively the same. The apparent difference between them is really one of perceptive processes.
4. The so-called threshold is not a true quantity. This may be shown by the same arguments that are applied to the so-called least noticeable difference.
5. If the estimate of the intensity of the stimulus may be considered as indicative of a corresponding increase in the intensity of sensation, this quantity increases much more slowly than the stimulus. The apparent rapid increase for very high intensities may be due to perceptive processes and not be a true increase in sensation.
6. The variation of the probable and constant errors renders inexact the use of the probability integral in the method of right and wrong cases.
7. The variations in the confidence of observers and in the percentage of right cases in guessing, goes to prove that there is no such quantity as a least noticeable difference.
8. The accuracy of discrimination is in general probably independent of the place of stimulation, except for very low intensities, which have less intensive effect at some places than at others. Practice seems to aid the discrimination at places not accustomed to pressure stimuli.

9. The intensive effect of impact stimuli for pain is equally dependent upon the mass and the square of the velocity. The intensive effect for such stimuli causing only impact sensations apparently increases faster for the velocity than the mass, but more slowly for the square of velocity than for the mass. If this be true, the stimulus for impact sensations is different from that for pain sensations, the stimulus for pain being mv^2 and that for impact sensations being mv^k, in which k is somewhat greater than unity, and possibly subject to individual variations.
10. The intensity of dermal sensations, is inversely related to the area of stimulation. If we assume any of the psycho-physical laws deduced, the intensity of the sensation increases much more slowly than the reciprocal of the logarithm of the area.
11. The intensive effect of the area may be explained by the probable physiological process of stimulation, the effect upon the sensory nerves being dependent upon the energy expended upon the surrounding tissues. The stimulus in pressure sensations is not to be considered the force applied, but the work done by this force, or more strictly the energy lost by the mass applied.
12. The intensity of pressure sensations decreases for low intensities with the time of pressure. For high intensities causing pain, the intensity of the sensation of pain increases with the time of pressure. The relation of the intensive effects of the time and that of the intensity of stimulation for pain sensations is probably that of a direct proportion.
13. The time phenomena of dermal stimulation support the theory advanced as to the process of stimulation. They also tend to show that dermal pain is a distinct sensation.

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